

Bridging knowledge and practice: Prospective mathematics teachers' analysis of student worksheet

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Abstrak Memahami kesulitan siswa dalam memecahkan masalah matematika sangat penting untuk pengajaran yang efektif. Penelitian ini bertujuan untuk mengetahui kemampuan calon guru matematika dalam menganalisis hasil kerja siswa dalam menyelesaikan soal-soal PISA, dan mengidentifikasi jenis bantuan yang disarankan untuk mengatasi kesulitan siswa. Dengan menggunakan pendekatan deskriptif kualitatif dengan jenis studi kasus, subjek penelitian ini terdiri dari dua orang calon guru matematika dari salah satu universitas di Aceh, yang memiliki pengetahuan yang baik dalam menjawab soal-soal PISA. Data dikumpulkan melalui tes tertulis, wawancara, dan dokumentasi hasil analisis untuk jawaban siswa. Penelitian ini dilakukan dalam dua tahap: (1) subjek menganalisis, menilai, dan merekomendasikan bantuan untuk hasil kerja siswa; dan (2) peneliti mengevaluasi hasil analisis dan kesesuaian bantuan yang diusulkan subjek. Temuan menunjukkan bahwa kemampuan *Pedagogical Content Knowledge* (PCK) guru yang bervariasi dapat mempengaruhi ketepatan analisis dan efektivitas bantuan yang direkomendasikan. Subjek dengan PCK yang tinggi dapat menganalisis hasil kerja siswa dan menyarankan bantuan dengan tepat. Sebaliknya, Subjek dengan PCK yang rendah hanya menganalisis dan memberikan saran untuk mengatasi kesulitan siswa secara umum. Studi ini menekankan pentingnya mengintegrasikan pengetahuan konten dan pedagogi dalam mempersiapkan calon guru.

Kata kunci *Pengetahuan konten, Pengetahuan pedagogis, PISA, Calon guru matematika*

Abstract Understanding students' difficulties in solving mathematics problems is crucial for effective teaching. This study aims to determine the ability of prospective mathematics teachers to analyze students' work in solving PISA problems and identify the types of assistance recommended to overcome students' difficulties. Using a descriptive qualitative design within a case study framework, the research subjects consisted of two prospective mathematics teachers from a university in Aceh, who had good knowledge in answering PISA problems. Data was collected through written tests, interview, and documentation of analysis results for student answers. This study was conducted in two stages: (1) the subjects analyzed, assessed, and recommended assistance for students' work, and (2) the researcher evaluated the results of the analysis and the appropriateness of the assistance proposed by the subjects. The findings indicate that varying Pedagogical Content Knowledge (PCK) abilities of teachers can affect the accuracy of the analysis and the effectiveness of the recommended assistance. Subjects with high PCK can analyze students' work and suggest assistance appropriately. In contrast, subjects with low PCK only analyze and provide suggestions to overcome students' difficulties in general. This study emphasizes the importance of integrating content knowledge and pedagogy in preparing prospective teachers.

Keywords *Content knowledge, Pedagogical knowledge, PISA, Prospective mathematics teacher*

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Introduction

The Ministerial Regulation on Primary and Secondary Education (Permendikdasmen) Number 10 of 2025 emphasizes that high school graduates must possess eight dimensions of graduate profiles. Among these are critical reasoning, creativity, and communication. The development of these competencies largely depends on the quality of instruction, particularly the teacher's Pedagogical Content Knowledge (PCK). PCK, as introduced by Shulman (1986), represents the integration of subject matter knowledge with pedagogical strategies, enabling teachers to effectively convey complex concepts to students. This intersection between content expertise and instructional practice is further elaborated in conceptual models such as that of Sorto, Marshall, Luschei, and Camoy (2009), which build upon teacher preparation theories. These models highlight the importance of teacher capabilities and instructional practices (Shulman, 1986; Silver, Stein, Smith, & Henningsen, 2000; Ball & Bass, 2000) in shaping meaningful student learning experiences.

PCK represents a teacher's capacity to integrate subject matter expertise with effective teaching practices, acknowledging that content knowledge is inherently tied to the learning process (Cochran, DeRuiter, & King, 1993). In Indonesia, teacher competencies are also mandated to reflect this integrated approach. According to Law No. 14 of 2005 on Teachers and Lecturers, teachers must demonstrate four core competencies: pedagogical, professional, personal, and social competencies. These competencies collectively establish a comprehensive framework for effective teaching, enabling educators to meet both subject-specific and student-centered learning objectives.

In order to effectively prepare the future teachers, particularly in mathematics education, educational institutions must focus on developing both PCK and subject matter expertise. Shulman (1987) introduced the concept of PCK, emphasizing that effective teaching requires more than just mastery of the subject matter. Teachers must also understand how to present the material in ways that are accessible to students, bridging the gap between content knowledge and pedagogical strategies. Such integration of knowledge is crucial for fostering students' deep understanding of mathematical concepts.

In line with this, one of the goals of the Mathematics Education Program at the university level is to produce prospective teachers who can understand their students in depth. This deep understanding enables teachers to implement learning strategies that are not only effective but also efficient and targeted. By fostering this capability, the program ensures that future educators are prepared to create learning environments where all students can thrive, a critical component in achieving high-quality education (Shulman, 1987). This emphasis on developing a comprehensive skill set for teaching mathematics mirrors global standards and prepares candidates to meet the increasing demands of contemporary education systems.

One of the effective methods for assessing prospective teachers' capabilities is through evaluating their skills in analyzing students' work. This approach provides insight into students' cognitive processes and problem-solving strategies. By understanding students' thought processes, prospective teachers can offer tailored support for those who face challenges or obstacles in problem-solving. This approach aligns with the objectives of the "Teaching Mathematics I" course within the Teacher Education International Standard program at one of the universities in Aceh, which emphasizes developing future teachers' abilities to identify and address diverse student learning needs. The course is designed to equip students with the skills to design instructional processes, evaluate learning outcomes, and implement remedial strategies

to address gaps in the learning experience. Enrollment in this course requires successful completion of the Learning Analysis course, ensuring that students possess the foundational skills necessary for effective assessment and evaluation of learning outcomes. Consequently, students in this course are expected to critically assess and analyze learning results, enabling them to enhance instructional quality and support diverse learner needs.

Existing literature reveals a discrepancy between teachers' recognition of the value of critical thinking and their actual understanding and preparedness to teach this skill in classroom practice (Anggraeny & Khongput, 2022; Kanmaz, 2022; Munkebye & Gericke, 2022; Yuan & Stapleton, 2020; Yuan et al., 2022). Otherwise, recent studies indicate that preservice teachers' understanding of non-routine problems improves when teacher education programs emphasize critical thinking, case-based reasoning, and engagement with complex, unfamiliar situations. Such training supports deeper problem interpretation and flexible reasoning, which are essential for addressing non-routine tasks in instructional practice (Christodoulou & Papanikolaou, 2023; Staberg et al. 2026). This suggests that teacher education programs should integrate problem-solving skills across all aspects of training, from curriculum development to classroom management.

In light of these findings, it is clear that developing prospective teachers' problem-solving abilities is not only crucial for their professional growth but also for enhancing the problem-solving capacity of their students. This underscores the need for continuous improvement in teacher training programs to ensure that future educators are fully equipped to meet the challenges of contemporary education.

The goal of this study is to assess the ability of prospective mathematics teachers when analyzing junior high school students' work in solving PISA problems and to identify the types of support that are recommended to address student difficulties based on the components of PCK. This research is essential because prospective teachers must not only possess content knowledge but also be capable of applying pedagogical strategies to help students navigate complex, real-world problems, such as those found in PISA assessments.

Finally, this study aims to bridge the gap between theory and practice in teacher education by equipping prospective teachers with the tools to analyze and address student difficulties effectively. By fostering a deep understanding of both content and pedagogy, teacher education programs can better prepare future educators to raise student performance not only in national contexts but also in global assessments like PISA.

Prior research

Ball, Thames, and Phelps (2008) argue that effective mathematics teaching requires teachers to not only know the mathematical content but also anticipate potential student difficulties, misconceptions, and alternative problem-solving strategies. This is where PCK becomes particularly significant, as it equips teachers with tools to diagnose learning challenges and adapt their instruction accordingly. Mathematics education programs, therefore, need to provide prospective teachers with both deep mathematical knowledge and the ability to teach that knowledge effectively. This dual focus helps future teachers develop a professional competence that improves student learning outcomes (Hill, Rowan, & Ball, 2005).

Research by Darling-Hammond and Bransford (2005) supports the idea that teacher education programs should integrate subject-specific pedagogy, allowing future educators to develop both the theoretical and practical skills necessary for classroom success. In mathematics

education, this means that future teachers must be well-versed in both pure mathematics and the pedagogical strategies tailored to teaching mathematics effectively. Studies have shown that teachers with a strong PCK not only deliver more effective lessons but also foster higher levels of student achievement, particularly in complex subjects like mathematics (Hattie, 2008). Given the complexity of teaching mathematics, the designed programs to educate prospective mathematics teachers must prioritize both content mastery and pedagogical development. By doing so, these programs ensure that future teachers are equipped to meet the diverse needs of students and contribute to improved educational outcomes in mathematics.

In addition to pedagogical knowledge, content knowledge plays a crucial role in the teaching-learning process. As highlighted by Shulman (1986), the integration of both pedagogical and content knowledge is necessary to address the complexities of classroom teaching. Content knowledge, often referred to as subject matter knowledge, is the foundation upon which teachers build their pedagogical strategies. Koehler, Mishra, and Cain (2017) emphasize that content knowledge is the knowledge about the actual subject matter that is to be learned or taught, and it is essential for teachers to not only know the content but also understand how to convey it effectively to students. Several studies have shown students' weaknesses in understanding mathematical concepts. The study by Kholid, Rofi'ah, Ishartono, Waluyo, Maharani, Swastika, Faiziyah, and Sari (2022) concluded that students have difficulty in mathematization. Students at low ability levels demonstrate basic, although incomplete, problem-solving strategies (Lestari, Najih, Putri, Murtafiah, Yahya, & Suwarno, 2025). This can provide teachers with a foundation for improving the learning process by ensuring proper conceptual mastery.

According to Ball et al. (2008), strong content knowledge allows teachers to anticipate common misconceptions students may have and develop strategies to address them. This ability to predict challenges in student understanding is part of what makes a teacher highly effective. Teachers with deep content knowledge are also more likely to make connections between different concepts, making learning more meaningful and comprehensive for students (Hill et al., 2005). Moreover, they are better equipped to adapt their teaching methods to accommodate diverse learners, ensuring that the content is accessible to all students.

Research further suggests that professional competence, which includes both pedagogical and content knowledge, is positively correlated with student outcomes. Hattie (2008), in his meta-analysis, concluded that teacher expertise is one of the most significant factors influencing student achievement. Teachers who possess both content knowledge and PCK are better able to implement instructional strategies that enhance learning, leading to improved student performance. This highlights the critical role that content knowledge plays in supporting a more effective learning process. Therefore, teacher education programs must emphasize the development of content knowledge alongside pedagogical skills to prepare future educators for the complexities of modern classrooms.

In addition to possessing both pedagogical knowledge and content knowledge, prospective teachers must also develop PCK, a unique combination of these two elements that enhances their ability to teach effectively. Shulman (1986) introduced PCK as the knowledge that enables teachers to transform subject matter into comprehensible content for students. This transformation is particularly crucial in mathematics, where abstract concepts often present challenges to students. PCK helps teachers identify the most effective ways to explain these concepts, making them accessible to learners of varying abilities.

Sorto et al. (2009) note that PCK manifests in the classroom through various practices, as it involves the application of mathematical knowledge in teaching others. This means that teachers not only need to know mathematics but must also understand how to structure and present mathematical ideas in ways that promote understanding. PCK is therefore vital for addressing common student misconceptions and for guiding students through problem-solving processes.

Hill et al. (2005) also emphasize that PCK is developed through specialized training and practice. Teachers build this knowledge by engaging in teaching experiences where they learn how to anticipate student difficulties, scaffold learning, and design instructional strategies that enhance student comprehension. This aligns with the view of Hill and Ball (2004), who suggest that the variety of skills necessary for effective teaching are acquired through deliberate practice and focused professional development. Teachers with strong PCK are better able to adapt their instruction to meet the needs of diverse learners, facilitating improved student outcomes in mathematics education.

The importance of developing PCK in teacher education programs cannot be overstated. By equipping future teachers with the ability to integrate content knowledge with pedagogical strategies, educational institutions help create educators who are not only subject matter experts but also skilled in communicating complex ideas effectively. This holistic approach to teacher preparation ultimately contributes to higher levels of student achievement, especially in challenging subjects like mathematics.

As Hill and Ball (2004) note, effective mathematics instruction requires a deep understanding of both content and pedagogy. Teachers must not only grasp mathematical concepts but also know how to teach these concepts in a way that is accessible and meaningful to students. This ability to bridge theory and practice is critical to the success of preservice teachers. The development of these skills is further supported by research suggesting that high-quality teacher preparation programs contribute to improved student outcomes by equipping teachers with the necessary tools to tailor instruction to diverse learning needs (Darling-Hammond, 2010).

Methods

This study employed a descriptive qualitative design within a case study framework, allowing for an in-depth and contextualized exploration of prospective mathematics teachers' understanding and analysis of students' worksheets. As noted by Creswell and Poth (2018) and Merriam and Tisdell (2016), qualitative approaches are effective in uncovering instructional challenges and evaluating teaching effectiveness within specific educational contexts. This design allowed for an in-depth exploration of participants' pedagogical reasoning and content knowledge as reflected in their worksheet analyses.

Participants

The participants in this study were 7 undergraduate students from the Mathematics Education Program at a university in Aceh, Indonesia, who were enrolled in the course Teaching Mathematics I (Mathematics Lesson Planning). Two participants from the course were designated as Subject X and Subject Y. They were selected from seven students who took the PISA-based mathematics problem-solving test, and they provided accurate and reasoned solutions in solving the problem. Next, the subjects were asked to carry out analysis and assessment of the students' work results. Subject X demonstrated a high level of mathematical

proficiency and exhibited strong pedagogical content knowledge (PCK), offering highly accurate and targeted instructional support. In contrast, Subject Y displayed a moderate level of PCK, with suggestions that were applicable only to parts of the student work and were generally basic in nature.

Instrument

The instrument includes: (a) PISA problems and answer key. The PISA 2012 tasks emphasize mathematical literacy in real-life contexts, focusing on formulating, employing, and interpreting mathematics. The study aims to assess students' problem-solving skills in depth. So, PISA 2012 is more suitable than PISA 2018, which gives greater attention to adaptive reasoning within digital contexts. The instrument incorporates test items from PISA level 6 from the 2012 assessment. The title is "Store Ice Cream" with space and shape content, and occupation context, aims to find many circles that can be arranged on a plane to follow the rules and use the scale (OECD, 2012). (b) Responses from junior high school students. All student answer sheets were collected, and responses were selected based on criteria that included misconceptions, errors in solution methods, diversity in solution approaches, and the strategies employed by students. (c) Prospective teacher responses and interview. The work completed by prospective teachers in solving the PISA tasks accurately demonstrates their content knowledge and comprehension of PISA's contextual demands, qualifying them for inclusion in the research. (d) Analysis of student and prospective teacher work using rubrics.

An assessment rubric was employed based on PCK components developed by Baker and Chick (2006), Koirala, Davis, and Johnson (2008), and Depaepe, Verschaffel, and Kelchtermans (2013). The rubric in Table 1 was used to assess students' and prospective teachers' answers to PISA questions. Next, the rubric in Table 2 was used by researchers to analyze the assessments carried out by prospective teachers on student work results. The stages of data analysis in the field were guided by the model proposed by Miles, Huberman, and Saldana (2014), which includes data reduction, data presentation, and verification or decision-making.

Table 1. Assessment rubric for student and prospective teachers' work on PISA problems (Sulastri et al., 2019)

Aspect of analysis	Scale 4	Scale 3	Scale 2	Scale 1
Accuracy in using mathematical concepts	Uses appropriate mathematical concepts correctly and consistently across all parts of the problem	Uses appropriate mathematical concepts, but only for a small part of the problem	Uses mathematical concepts with some inaccuracies and inconsistencies related to the problem	Uses incorrect or inappropriate mathematical concepts that do not align with the problem
Accuracy of computation	All answers are correct, detailed, and in logical order	All answers are correct but lack detail and order	Most answers are correct but lack detail and are not in logical order	Only a few answers are correct, and they are neither detailed nor logically ordered

Aspect of Analysis	Scale 4	Scale 3	Scale 2	Scale 1
Identifying steps in problem solving	Accurately identifies every step in solving the problem	Accurately identifies most steps in solving the problem	Accurately identifies only a few steps in solving the problem	Identifies only known or prompted steps, or provides only the final result without process
Providing reasoning or ideas	Provides reasoning or ideas for each step from the beginning to the end of the solution	Provides reasoning or ideas for most steps of the solution	Provides reasoning or ideas for only a few steps	Does not provide reasoning or provides only brief or unclear reasoning

Table 2. Assessment rubric for the result of prospective teachers' analysis (Sulastri et al., 2019)

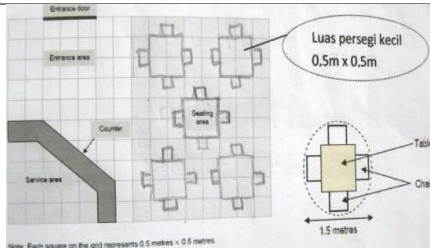
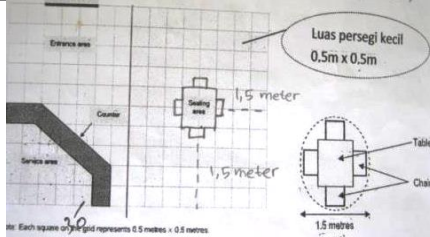
Aspect of analysis	Scale 4	Scale 3	Scale 2	Scale 1
Accuracy of analysis conducted on student work	Accurately analyzes all aspects of student work	Accurately analyzes most aspects of student work	Accurately analyzes a few aspects of student work	Analyzes most aspects inaccurately or incompletely
Ability to understand students' thinking and identify misconceptions	Accurately identifies and understands all conceptual errors in student work	Accurately identifies most conceptual errors in student work	Accurately identifies a few conceptual errors in student work	Identifies a few conceptual errors in student work but inaccurately
Ability to evaluate based on rubric criteria	Provides accurate and appropriate evaluation for each rubric aspect	Provides accurate evaluation for most rubric aspects	Provides accurate evaluation for a few rubric aspects	Provides evaluation for a few aspects but inaccurately or inconsistently
Accuracy of suggested strategies/support	Recommends appropriate and relevant support/strategies for all aspects of the problem	Recommends appropriate support/strategies for most aspects of the problem	Recommends somewhat appropriate support/strategies for a few aspects of the problem	Recommends inappropriate or irrelevant support/strategies for the problem

Findings and Discussion

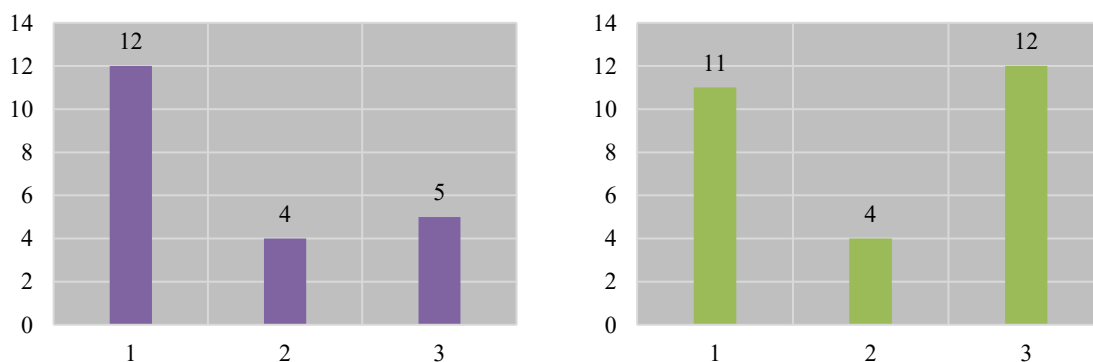
Students' works on PISA problem

Table 3 displays sample worksheets from three junior high school students. These answers were chosen for inclusion in the analysis. Each evaluation made by Subject X and Y on these sample worksheets serves as main data throughout the study.

Table 3. Samples of students works on ‘Ice Cream Shop’ problem

<p>(1)</p>	<p>Pendapat tentang Soal: Soalnya Luasnya Membingungkan</p> <p>Penyelesaian: Dik = Luas persegi kecil = $0,5m \times 0,5$ Diameter lingkaran = $1,5m$ Dit = jumlah susunan meja dan kursi .. ? Jwb = Luas area seating = $36 m^2$ caranya = jumlah persegi kecil = $72 \times 0,5 = 36 m^2$ Luas lingkaran = $\pi d = 3,14 \times 1,5 = 4,71 m^2$ Jadi, jumlah banyaknya seating maksimal ialah $\frac{36 m^2}{4,71 m^2} = 7,6$ buah (7 jumlah banyak susunan dan 6 jarak tiap antara susunan lain).</p>
<p>(2)</p> <p>Pendapat tentang Soal:</p> <p>Penyelesaian: Jumlah maksimum Susunan yang dirancang oleh pemilik toko Untuk Seating area ditokonya adalah 5 susunan.</p>	
<p>(3)</p> <p>Pendapat tentang Soal:</p> <p>Penyelesaian: Jumlah maksimum yang dapat dirancang adalah 1. Karena jarak antara dinding 0,5 dan dari susunan beda lain 0,5, Dan setiap meja harus memiliki 4 Kursi. Jarak setiap kotak adalah 8 kotak dengan syarat: $\frac{0,5}{1,5} \times 8$</p>	

Graph 1 shows the assessment outcomes produced by Subject X and Y on three students’ worksheets according to the rubric in Table 1. Subject X awarded the student work (1) a score of 12, noting that the student understood the problem but was insufficiently attentive to the rules specified in the task. For student work (2), a score of 4 was given because the student-focused primarily on the design illustrations without considering both required rules and provided no explanation, making it difficult to interpret the student’s reasoning, which appeared to prioritize visual over analytical reasoning. Student work (3) received a score of 5, as the student understood the problem but did not apply the correct approach in creating a variety of possible designs.



Graph 1. Scores given by Subject X (orange) and Y(black) for students’ work (1), (2) and (3)

In evaluating student work (1), Subject Y awarded a score of 11, noting accurate and detailed calculations for most of the task, though identifying steps and providing reasoning were limited. For student work (2), Subject Y gave a score of 4, as all aspects received low ratings due to the focus solely on design illustrations, with no attention to the given rules, making it difficult to interpret the student's problem-solving approach. Student work (3) received a score of 12, as it demonstrated accurate concepts and calculations for most parts of the task, though steps for resolution were only partially identified, and reasoning was appropriately applied.

Prospective mathematics teachers' performance

The work of subject X and Y on the 'Ice Cream Shop' problem can be seen in Table 4. Subject X's work on the 'Ice Cream Shop' problem demonstrates a clear understanding of the problem and uses the specified rules without applying any formula to determine the number of possible seating designs, leading to the correct answer. According to the assessment rubric, Subject X received a score of 11, as the explanations and ideas presented were incomplete, and the steps identified to solve the problem were not entirely comprehensive. Subject X self-assessed with a score of 14, recognizing weaknesses in reasoning and calculation.

Subject Y's work demonstrates an understanding of the issues presented in the task, enabling them to design multiple table arrangements correctly for the seating area. However, a notable weakness is the lack of explanation for each stage of the design process, particularly regarding the number of tables required according to the problem's rules. According to the rubric, Subject Y receives a score 9, as no explanations or ideas were provided in the problem-solving process, and steps for solving the problem were not identified, nor were any calculations shown. In self-assessment, Subject Y self-assessment is 16, indicating a disregard for several aspects of the assessment rubric.

Table 4. Subject X and Y works on 'Ice Cream Shop' problem

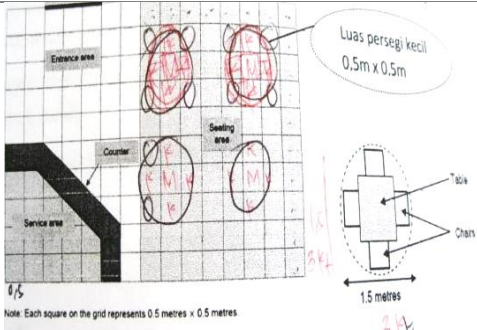
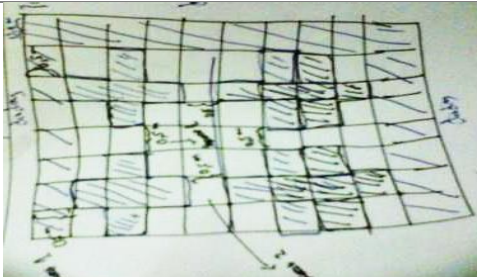
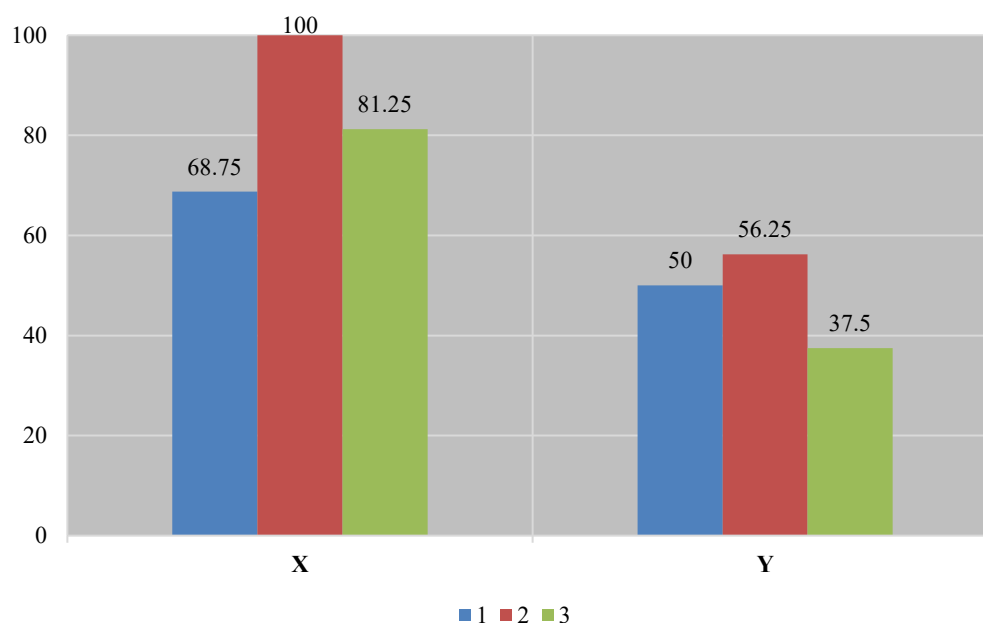
<p>X</p>	<p>Pendapat tentang Soal: Soal ini membantu siswa berpikir karena kebanyakan siswa menggunakan rumus padahal dapat dijangkau hanya dgn menghitung kotak</p> <p>Penyelesaian:</p> <p>Hanya 4 susunan kursi</p> <p>$8 \times 10 = 80$ kotak $0,5 \text{ m}$ $= 40 \text{ m}^2$</p> <p>1 susunan kursi memerlukan $3 \times 3 = 9$ kotak $0,5 \text{ m}$ $= 4,5 \text{ m}^2$</p> <p>Banyak susunan kursi = $\frac{80}{9} = 8,88$</p> <p>ditambah spasi per kursi $0,5 \text{ m}$ atau 1 kotak.</p>	
<p>Y</p>	<p>Aturan 1 : setiap susunan harus disusun $\geq 0,5 \text{ m}$ dan $\geq 0,5 \text{ m}$</p> <p>Aturan 2 : Jarak antara susunan \geq selitiran $0,5 \text{ m}$ dan susunan lain.</p> <p>Jadi jumlah susunan paling mungkin 4 macam.</p>	

Table 5 presents an example of assessment results for Subject X upon analyzing students' work based on the rubric presented in Table 2. As seen, X receives a value 68.75 indicating a good level of analytical ability. The overall assessment for the two subjects' analytical abilities is summarized in Graph 2. There one can see that in case of students' work (2) and (3), X receives a value 100 and 81.25 signifying a very high level of analytical proficiency and a good level of analytical ability, respectively. Subject Y's ability to analyze student work (1) and (2) scored 50 and 56.25, respectively, indicating a sufficient level of analytical skill. For student work (3), the score was 37.5, suggesting that Subject Y's analytical skills were less effective.

Table 5. An example of assessment for Subject X while analyzing student worksheet (1)

No	Aspect	Weight	Scale	Scale description	Score
1	Assessment of the accuracy of the analysis carried out on student work results	2×	3	Assess most aspects of the analysis correctly	6
2	Ability to understand students' thinking about misconceptions	1×	2	Able to recognize a small number of conceptual errors in students' work accurately	2
3	Ability to give assessments according to the rubric	1×	3	Able to provide accurate assessment of most aspects	3
Total score					11
Maximum score					16
Value = (Total score/Maximum score) × 100					68.75



Graph 2. Prospective teachers' ability in analyzing student worksheets 1 (blue), 2 (red), and 3 (green)

The findings are further supported by the interview scripts between researcher (P) and subject X and Y as presented below.

- P : *After analyzing the students' work, what are your thoughts on the students' work and their thinking skills in solving the problems?*
- X : *In my opinion, it turns out that the students themselves don't fully understand what PISA actually is or how to answer the problems. So, they're too focused on the problems in school, even though the problems are supportive. The students are generally lacking in ability. Their thinking skills are still below average. So, we need to teach them from the basics.*
- Y : *Student (1)'s reasoning is quite good. He concluded that to arrange each table, he first found the area of the table, then the area of the seating area, and then divided it. Divide the area of the seating area by the area of the table. The others went straight to the answer.*
- P : *How does your work compare with the students' work? Did the students' work help you in solving the problems?*
- X : *Some of the answers were close. For some, they were the same, but the method was different. I didn't write much, while the students wrote a lot.*
- Y : *There were many differences in the mathematical modeling. Most of the students' answers were incorrect. Some were close to the correct answer, but some were inaccurate in the circle formula. The students' answers weren't helpful because they went straight to the results without a bridge.*
- P : *What kind of assistance is recommended for each student experiencing difficulty solving PISA questions? Why did you choose that assistance?*
- X : *The explanation really depends on how the teacher explains it. We can manipulate misconceptions in other ways. Perhaps we can use this formula, or we can create a drama. We can make props out of ceramic tiles on the classroom floor. We can count the number of tiles equal to this, and then we can organize them into groups. It does require many classes for groups, but from those classes, we can help the children understand the problem. The class can be a prop.*

The interview highlights that student generally struggle with PISA-type problems due to limited conceptual understanding and below-average thinking skills, often relying on familiar school problem formats. Most students skipped critical reasoning steps, used incorrect formulas, or went straight to answers, making their work of little use in guiding teachers' problem-solving. Only a few students demonstrated structured, logical reasoning. To address these difficulties, the prospective mathematics teachers suggested using concrete, hands-on, and contextual strategies such as manipulatives, visual demonstrations, or concrete model to gradually build understanding and correct misconceptions over multiple lessons.

In evaluating the appropriate instructional support, Subject X received a score of 100 (very good), particularly for recommending the use of tangible learning aids such as classroom floor tiles and for encouraging students to work collaboratively to deepen their understanding of the problem. This form of support is consistent with Akkoc and Yesildere's (2010) assertion that prospective teachers' awareness of students' learning difficulties is shaped by their instructional practices. Such support is also identified as beneficial for addressing challenges students commonly encounter when engaging with PISA tasks. Overall, Subject X's PCK aligns with the

components outlined by Baker and Chick (2006), Koirala et al. (2008), and Depaepe et al. (2013), placing it within a strong proficiency category.

To further contextualize the relevance of Subject X's PCK in addressing student difficulties, recent studies highlight the central role of PCK in enhancing teaching effectiveness. Krauss et al. (2008) demonstrate that well-developed PCK improves teachers' ability to analyze student understanding and adapt their instructional approaches accordingly. Intervention studies show that domain-specific PCK enhances preservice teachers' ability to focus on relevant task features and accurately judge the difficulty of problems that typically elicit student errors, suggesting that well-developed PCK contributes to more effective identification of student misconceptions in instructional practice (Grigaliūnienė et al., 2025). Additionally, systematic reviews of mathematics education research highlight that PCK encompasses knowledge of student misconceptions and related pedagogical strategies, which are critical when addressing conceptual misunderstandings in challenging content domains (Brunner et al., 2024). Depaepe et al. (2013) also emphasize that the use of concrete instructional tools, evident in Subject X's approach, supports students' comprehension of complex concepts. Such tangible media foster engagement and help students overcome cognitive obstacles, aligns with the findings of Akkoc and Yesildere (2010). Furthermore, Taber (2014) show that reflective practice and systematic analysis of student work are vital for effective PCK development, further supporting the placement of Subject X's performance within a good PCK category.

In contrast, Subject Y received a score of 50 (sufficient) for suggesting instructional aids, as the support provided which focus only on reinforcing a limited portion of the problem and remained largely general. This corresponds with Isiksal and Cakiroglu's (2010) observation that efforts to strengthen conceptual understanding and relational reasoning must go beyond repetition; instead, contextual instructional aids are necessary to enhance students' grasp of mathematical ideas.

Analysis of Subject Y's performance indicates that more nuanced instructional support, particularly through contextualized tools and targeted scaffolding, is essential for advancing student comprehension and promoting higher-order thinking. These findings highlight the need for ongoing professional development that equips teachers with strategies adaptable to diverse learner needs.

In relation to the Subject Y's analytical abilities, prior research further emphasizes the importance of effective instructional strategies and support mechanisms. Subject Y's scores of 50 and 56.25 for analyzing student work (1) and (2) indicate an adequate but improvable level of analytical skill, whereas the score of 37.5 for student work (3) suggests difficulty in offering comprehensive analytical support, which is crucial for facilitating deep learning. Schmidt, Wang, and McKnight (2005) argue that teachers must employ varied strategies tailored to students' needs to cultivate higher-order thinking, aligning with the observation that Subject Y's instructional suggestions were overly general and insufficiently aligned with the specific demands of the problems. Their work underscores the importance of scaffolding that provides focused support, gradually withdrawn as students develop proficiency.

Additionally, Koehler et al. (2014) highlight the value of contextual teaching aids that connect with students' real-world experiences, enhancing both engagement and conceptual understanding. Subject Y's score of 50 for proposing instructional aids therefore reflects a missed opportunity to integrate such resources more effectively. Van de Walle, Karp, and Bay-Williams (2020) further demonstrate that combining visual and tactile tools with traditional

instruction significantly strengthens students' comprehension and retention of mathematical concepts. Thus, while Subject Y exhibits adequate analytical ability, the inclusion of more diverse and contextually meaningful instructional aids would likely enrich students' learning and better support them in navigating conceptual challenges.

Conclusion

The findings of this study indicate that prospective mathematics teachers demonstrate varying levels of ability in analyzing student work based on the components of Pedagogical Content Knowledge (PCK). Some participants showed a strong capacity to accurately assess student responses, identify misconceptions, and provide appropriate instructional support aligned with the assessment rubric. This suggests a solid integration of both mathematical content knowledge and pedagogical understanding. However, other participants exhibited only a partial understanding of PCK, with limited accuracy in their analysis and a less consistent application of the rubric. These variations highlight the need for enhanced opportunities within teacher education programs to develop prospective teachers' ability to interpret student thinking and respond with effective instructional strategies. Strengthening PCK is essential to prepare future educators for diagnosing learning difficulties and supporting meaningful student learning in mathematics.

The results emphasize the necessity of integrating comprehensive PCK training into teacher education curricula. By prioritizing the development of both mathematical content knowledge and pedagogical skills, future educators can be better prepared to navigate the complexities of student learning and misconceptions. This holistic approach benefits not only teachers but also leads to improved student outcomes in mathematics.

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