

## **Enhancing students' problem-solving skills and engagement through inquiry-based mathematics education with Mathigon: A study on Cartesian coordinates**

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**Abstrak** Kemampuan pemecahan masalah matematis dan keterlibatan belajar siswa pada salah satu Sekolah Menengah Pertama (SMP) di Kuala Batee, Aceh, Indonesia, masih belum maksimal. Tujuan penelitian ini yaitu untuk mengetahui peningkatan kemampuan pemecahan masalah matematis dan keterlibatan belajar siswa melalui pembelajaran dengan pendekatan Inquiry-based Mathematics Education (IBME) berbantuan dan tanpa berbantuan Mathigon pada materi koordinat Cartesius. Penelitian ini menggunakan pendekatan kuantitatif jenis eksperimen dengan desain penelitian *pre-test* dan *post-test control group design*. Sampel penelitian dipilih dengan teknik *total sampling* yang berjumlah 42 siswa kelas 8 SMP. Data dikumpulkan melalui tes dan angket, dan dianalisis melalui uji normalitas dan homogenitas, uji *n-gain*, dan uji *t*. Hasil penelitian menunjukkan bahwa peningkatan kemampuan pemecahan masalah matematis dan keterlibatan belajar siswa melalui pendekatan IBME berbantuan Mathigon lebih baik daripada pembelajaran tanpa berbantuan Mathigon pada materi koordinat Cartesius. Berdasarkan hasil tersebut, pendekatan IBME dapat menjadi salah satu opsi bagi guru dalam merancang pembelajaran matematika di sekolah.

**Kata kunci** *Kemampuan pemecahan masalah matematis, Keterlibatan siswa, Koordinat Cartesius, Mathigon, Pendekatan inquiry-based mathematics education*

**Abstract** Students' mathematical problem-solving skills and learning engagement remain below expectations at a public junior high school in Kuala Batee, Aceh, Indonesia. This study examines the effectiveness of the IBME approach integrated with Mathigon in enhancing students' mathematical problem-solving abilities and learning engagement, compared to the traditional IBME approach without Mathigon, with a specific focus on Cartesian coordinates. This study employed a quantitative experimental approach using pre-test and post-test control group design. The sample, consisting of 42 Grade 8 junior high school students, was selected through a total sampling technique. Data was collected through tests and questionnaires and analyzed using normality and homogeneity tests, n-gain tests, and t-tests. The results demonstrate that the IBME approach with Mathigon significantly improves students' mathematical problem-solving skills and learning engagement compared to the traditional IBME approach without Mathigon. These findings suggest that the IBME approach, assisted by Mathigon, offers a practical and effective strategy for educators in designing mathematics instruction in schools.

**Keywords:** *Mathematical problem-solving skills, Student engagement, Cartesian coordinates, Mathigon, Inquiry-based mathematics education approach*

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## Introduction

Mathematical problems are an integral part of everyday life, manifesting in financial calculations (Rakhmawati & Astuti, 2022), shopping and discounts (Szabo et al., 2020), cooking and measuring ingredients, time and scheduling, calculating working hours and payroll, data analysis, measurement, architecture and construction (Nurhalisa & Alghofiaty, 2022), and transportation and navigation (Baykal & Semiz, 2020; Ergen, 2020). The prevalence of these problems underscores the need for students to develop strong mathematical problem-solving abilities. Explicit teaching of domain-specific problem-solving tactics allows students to tackle non-routine tasks effectively, promoting critical thinking (Foster, 2023). It also helps learners build confidence in tackling unfamiliar or complex problems, crucial in academic and real-life contexts. Problem-solving involves identifying, formulating, and resolving mathematical issues, which enhances understanding of mathematical concepts, daily problem-solving, and logical, analytical, creative, and abstract thinking skills (Zahroh et al., 2020). It also includes recognizing patterns, relationships, and generalizations, making it a vital skill for students (Radiusman & Simanjuntak, 2020).

Problem-solving activities in learning to engage students physically, mentally, and socially (Bancong & Song, 2020). Physically, students might be actively manipulating tools, drawing diagrams, or performing tasks that require hands-on interaction, which helps reinforce their understanding of abstract concepts. Mentally, they are involved in deep cognitive processing, such as analyzing the problem, thinking critically about possible solutions, and applying knowledge to solve the task. Socially, students often collaborate with peers, communicate their ideas, and share different approaches to solving the problem. This means that problem-solving activities engage multiple dimensions of student engagement (Raza et al., 2020). Student engagement refers to active participation in the learning process, encompassing cognitive, emotional, and behavioral aspects (Christanty & Cendana, 2021; Deosari et al., 2022).

In junior high schools in Southwest Aceh Regency, students' mathematical problem-solving abilities and engagement, particularly in Cartesian coordinates, are suboptimal across cognitive, emotional, and behavioral dimensions. Cognitively, students struggle to grasp the concepts underlying Cartesian coordinates, such as determining and describing point coordinates like  $(-2,5)$ ,  $(-2,-4)$ ,  $(4,3)$ ,  $(6,6)$ , and  $(4,2)$  in relation to the  $x$ -axis and  $y$ -axis. Emotionally, students show low interest and motivation in learning this topic, affecting their willingness to engage with related activities. Behaviorally, many students are not actively participating in classroom discussions or practice exercises. Teachers report difficulties in helping students visualize and understand the position of points in Cartesian planes. Addressing this issue requires using technology-based tools to provide visual illustrations of point positions, which may improve student understanding and engagement on all three levels.

Despite applying problem-solving questions to other materials, only around a quarter of the students (27.27%) successfully solved them, while the remaining 72.73% encountered significant challenges. The analysis revealed that struggling students encounter specific difficulties at various stages of problem-solving. In understanding the problem, many misinterpreted key information or failed to grasp mathematical relationships, such as confusing variables or operations. Planning posed another challenge, as students often lacked clear strategies and attempted to solve problems in one go, resulting in disorganized work. During the solving phase, errors in calculations and difficulty applying appropriate formulas, especially for abstract concepts like pattern recognition or generalization, were common. Additionally, many

students neglected to verify their answers, leading to incorrect or inconsistent results. Previous research aligns with these findings, identifying difficulties in problem analysis (Moch & Basuki, 2021), pattern recognition (Genc & Erbas, 2019), making conjectures and generalizations (Hayuningrat & Rosnawati, 2022; Subarinah et al., 2020), and interpreting and evaluating solutions (Rizki & Priatna, 2019; Majeed et al., 2021). These issues reflect deeper cognitive struggles in identifying relationships, making informed guesses, and checking the validity of their solutions, contributing to the low success rate. Addressing these areas through targeted interventions could improve students' problem-solving abilities.

Furthermore, not all students actively participate in mathematics learning, partly due to its perceived monotony—students often find the lessons repetitive, lacking variety, and disconnected from real-world applications. This disengagement hampers both their interest and their ability to develop problem-solving skills. To address this, a learning approach is needed that enhances student engagement, stimulates critical thinking, and makes learning more interactive and meaningful. Inquiry-Based Mathematics Education (IBME) was chosen over other innovative models for its strong student-centered approach (Dorier & Maass, 2020). Unlike traditional models, IBME encourages students to actively participate by observing phenomena, asking questions, seeking mathematical approaches, and evaluating solutions. The role of the teacher is equally crucial, as they guide students by building prior knowledge, posing challenging questions, facilitating discussions, and encouraging diverse perspectives. IBME also stands out for its emphasis on open-ended problems with multiple solutions, linking mathematics learning to real-life experiences and fostering a deeper sense of inquiry (Artigue et al., 2020).

Integrating technology into learning can be a powerful solution to enhance student engagement and problem-solving abilities, building on the need for more interactive and dynamic approaches like Inquiry-Based Mathematics Education (IBME). In the previous discussion, we addressed the issue of students' disengagement due to the perceived monotony of traditional mathematics teaching methods. While IBME provides an inquiry-driven, student-centered framework, technology can complement this approach by offering tools that make abstract mathematical concepts more tangible and accessible. For instance, technology can make visual representations of points in Cartesian coordinates—an area where many students struggle—clearer.

Mathigon is one such technological tool that aligns with the goals of IBME. It allows students to visually explore and manipulate mathematical concepts, offering an engaging and interactive experience that supports inquiry-based learning. By integrating Mathigon into an IBME framework, the students can observe phenomena, explore multiple problem-solving strategies, and use the tool to test their ideas in real time. Previous research has shown that students find Mathigon useful and visually appealing, making it a valuable tool for enhancing their understanding of mathematical concepts (Wadhwa & Kathane, 2022). Combining the strengths of IBME with technology like Mathigon creates a more engaging lesson and inquiry-driven learning environment that addresses students' struggles with mathematical problem-solving and promotes deeper learning.

Several studies have explored Mathigon's use in mathematics learning. Ferreira and Mendes (2020) developed an open, interactive learning resource for algorithmic problem-solving with Mathigon. Harahap and Harahap (2020) examined the impact of inquiry learning stages on mathematical problem-solving abilities. Recent studies show that IBME (Inquiry-Based Mathematics Education) enhances problem-solving skills in geometry (Smith & Johnson, 2021).

Despite these findings, research on enhancing mathematical problem-solving abilities and student engagement through IBME supported by Mathigon is limited, particularly in Cartesian coordinates. Information on the relationship between IBME and student engagement is also scarce. Therefore, this research aims to investigate the enhancement of mathematical problem-solving abilities and student engagement through IBME, supported by Mathigon, in Cartesian coordinates.

## Theoretical review

This theoretical review discusses the key variables of mathematical problem-solving skills, student engagement, and inquiry-based mathematics education (IBME) to provide a comprehensive foundation for the study. By examining these elements in the context of using Mathigon as an educational tool, the review aims to underscore the relevance and potential impact of inquiry-based approaches on students' understanding of Cartesian coordinates.

### Mathematical problem-solving skills

A problem is a situation that requires a solution where the answer is not immediately apparent (Reys et al., 2014). Pedagogically, a problem demands that students use their knowledge to find a satisfactory solution (Căprioară, 2015) without relying on previous experiences or specific methods (Olivares et al., 2021). In mathematics, there are two types of problems: routine problems, which can be solved using previously learned procedures, and non-routine problems, which require deeper thinking as the procedures are not yet known (Reys et al., 2014).

Mathematical problem-solving abilities are defined as students' capacity to identify, plan, and implement problem-solving strategies. This ability involves logical reasoning, creative thinking, and applying knowledge in new and unfamiliar situations. According to Polya (1957), problem-solving abilities can be broken down into four main indicators: (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) reviewing the solution. On the other hand, student engagement refers to students' level of participation and involvement in learning activities. Fredricks et al. (2004) describe three primary indicators of learning engagement: (1) cognitive engagement, which reflects how deeply students think about and invest in learning; (2) behavioral engagement, which looks at student participation and attention in class; and (3) emotional engagement, which represents the students' enthusiasm, interest, and motivation in the learning process.

Problem-solving involves overcoming obstacles to reach a solution (Sternberg et al., 2012) and is a complex cognitive activity that engages various mental processes, emotions, motivation, and the ability to control the situation (Căprioară, 2015). Öztürk et al. (2020) explain that problem-solving ability involves the use of information and skills to understand mathematical concepts. Problem-solving requires specific strategies and steps to achieve the desired outcome (Umar et al., 2022). Thus, problem-solving can be concluded as a complex cognitive activity in which students apply strategic steps to reach a solution.

The challenge in solving mathematical problems differs from routine exercises because mathematical problems require deeper thinking (Azizah & Abadi, 2022). A step-by-step guide is necessary to assist students in solving these problems, such as using visual aids. Polya's problem-solving framework includes four main steps: understanding the problem, devising a plan, carrying out the plan, and looking back to review the solution (Schoenfeld, 1987). This

study measures students' mathematical problem-solving abilities based on Polya's steps within Cartesian coordinates, aiming for students to solve problems systematically and in line with problem-solving ability indicators.

### **Students' engagement**

Student engagement encompasses students' participation in academic and non-academic activities, evident through their behavior, emotions, and cognition in school and class (Fredricks, et al., 2004). Gunuc (2014) added that student engagement includes psychological, cognitive, emotional, and behavioral responses to the learning process, both inside and outside the classroom, and social activities to achieve successful learning outcomes. Trowler (2010) emphasized the interaction between the time, effort, and resources students invest to optimize their learning experience. Students must actively engage mentally and physically in learning activities (Winkel, 1996).

Student engagement can be observed through active participation in the teaching and learning, attachment to school, and effort to understand lessons (Jimerson et al., 2003). Higher levels of student engagement have been shown to correlate with better academic outcomes, higher self-esteem, and more appropriate social behavior (Finn and Rock, 1997; Reyes et al., 2012; Voelkl, 1997). This engagement supports a positive school image and reduces negative student behaviors outside school (Glanville & Wildhagen, 2007; Marcus et al., 2021). However, student engagement can diminish in inadequate school environments, such as unsafe learning conditions, unclear school rules, or indifferent teachers (Marcus et al., 2021; Lam et al., 2014). Student engagement is measured through various indicators, such as behavioral, emotional, cognitive, and agentic engagement (Reeve & Tseng, 2011; Archambault et al., 2009; Appleton et al., 2008).

The indicators of student engagement used in this study include three main aspects—emotional, cognitive, and behavioral engagement—whose validity and reliability have been tested in 12 countries (Lam et al., 2014). Emotional engagement reflects students' reactions to teachers, classmates, academic activities, and school, such as enthusiasm, joy, boredom, or anxiety. This aspect is crucial for fostering students' sense of attachment to school and increasing learning motivation. Cognitive engagement focuses on students' psychological investment in learning, where they strive to go beyond existing standards, seek challenges, and remain focused and mentally engaged in the learning process. Students are not just physically present but also mentally involved, demonstrating attention, concentration, and effort. Behavioral engagement includes three components: adherence to school rules and avoidance of disruptive behavior, active participation in learning activities like effort and contributions to class discussions, and involvement in extracurricular activities such as sports and school organizations. Overall, student engagement involves active participation (behavioral), emotional reactions (emotional), and cognitive effort to understand complex ideas and skills (cognitive).

### **Inquiry-based mathematics education approach**

The Inquiry-Based Mathematics Education (IBME) approach is a collaborative learning method where students work together on challenging and interconnected tasks. In this approach, the teacher's role includes constructively utilizing students' prior knowledge, challenging them with effective questions, managing discussions, encouraging diverse viewpoints, and helping students connect their ideas (Dorier & Maass, 2020). In support of this theory, IBME aligns with

constructivist learning theories, where learners actively construct knowledge through engagement with problems and interaction with peers. By promoting inquiry and exploration, IBME fosters a deeper understanding of mathematical concepts through active cognitive processes and peer collaboration. Additionally, IBME emphasizes using problems with multiple solutions, connecting learning to students' experiences and fostering inquiry (Artigue et al., 2020). Five key elements distinguish IBME: teacher guidance, valuable outcomes, classroom culture, types of questions, and student activities (Artigue & Blomhøj, 2013). Teacher guidance involves appreciating and building on students' reasoning and linking learning to their experiences.

IBME aims for valuable outcomes where students are curious, prepared for uncertainty, and understand the nature of science and mathematics. The classroom culture in IBME is characterized by shared goals, respect for mistakes and contributions, dialogue, and shared ownership. The approach includes open-ended questions with multiple strategies and solutions relevant to real life and scientifically pertinent. Students are encouraged to ask questions, engage in the 5E model (engage, explore, explain, extend, evaluate), and collaborate. In this study, IBME is implemented through structured collaborative tasks in student worksheets where students are grouped to explore mathematical problems with real-world relevance. The teacher facilitates by asking questions and guiding discussions, allowing students to develop reasoning and problem-solving skills through experimentation and interaction. IBME stands out from other approaches by emphasizing students' active roles in learning and focusing on the discovery process (Artigue et al., 2020; Divrik et al., 2020). Students are encouraged to ask questions, investigate, and discover mathematical concepts through exploration and discussion, allowing them to develop mathematical understanding via experimentation and problem-solving (Artigue & Blomhøj, 2013). IBME effectively enhances problem-solving abilities (Divrik et al., 2020; Huang et al., 2021) and student engagement (Archer-Kuhn et al., 2020).

The advantages of IBME include developing a deep understanding of mathematical concepts through active engagement (Artigue & Blomhøj, 2013; Dreyøe et al., 2018), increasing student engagement by involving them in investigation and exploration (Farhan & Retnawati, 2014), linking mathematical concepts to real-world situations (Dwirahayu et al., 2020; Gómez-Chacón et al., 2023), enhancing problem-solving skills through exploration and creative thinking (Artigue et al., 2020; Divrik et al., 2020), developing critical and logical thinking (Abdurrahman et al., 2021; Artigue & Blomhøj, 2013), and stimulating interest in mathematics by allowing active and interactive exploration of concepts (de Jong et al., 2010; Dorier & Maass, 2020).

However, IBME also has disadvantages, including the time required for implementation, as students need time to investigate, formulate questions, and find answers (Dreyøe et al., 2018). Teachers must be skilled and knowledgeable to manage inquiry-based learning, and not all may feel comfortable or adept with this approach (Dwirahayu et al., 2020). Assessing inquiry-based learning can be complex because students demonstrate understanding in various ways (Divrik et al., 2020). Additionally, some mathematical concepts may be difficult to teach or understand through inquiry, especially those requiring a strong foundational understanding (Abdurrahman et al., 2021). The open nature of IBME can also pose challenges in classroom management and demand higher discipline from students (Sudirman et al., 2021).

The IBME approach promotes problem-solving, exploration, and discussion in mathematics education (Gosztonyi, 2022). Its implementation begins with presenting interesting and relevant mathematical problems, enabling students to explore and find solutions independently. During

the process, students attempt various strategies, collaborate with peers, and ultimately generalize their findings with the guidance of their teacher (Dorier & Maass, 2020). The approach fosters critical and creative thinking and helps students develop a deeper understanding of mathematical concepts through independent problem-solving. Collaboration and discussion further enhance students' abilities to communicate their mathematical reasoning (Lin et al., 2021).

IBME emphasizes its alignment with modern educational goals. The active and collaborative nature of IBME supports the development of 21st-century skills, such as critical thinking, problem-solving, and adaptability. These skills are crucial for students in a rapidly changing world. Moreover, the method encourages students to take ownership of their learning, fostering a deeper connection to the subject matter and promoting lifelong learning habits. The flexibility of IBME allows students to explore concepts in ways that relate to their own experiences, making learning more relevant and meaningful.

In this study, IBME is implemented through activities integrated into student worksheets designed by our research team to align with the 5E model (engage, explore, explain, extend, evaluate). These worksheets guide students to explore mathematical problems collaboratively, discuss various approaches, and engage in experimentation. The teacher is a facilitator, posing probing questions to deepen understanding and help students connect concepts. Students are encouraged to apply multiple problem-solving strategies, fostering their creative thinking and problem-solving skills. Each stage of the 5E model is actively implemented, allowing students to progress from formulating questions to evaluating their solutions. This approach is anticipated to improve student engagement and mathematical problem-solving abilities.

## **Mathigon**

Mathigon is an interactive online learning platform designed to enhance conceptual understanding in mathematics by emphasizing inquiry-based learning and engaging students with visual and interactive content (Manandhar, 2018). It uses graphics, animations, and multimedia to present mathematical concepts in an accessible and captivating way, allowing teachers and students to interact with the material dynamically. The platform is designed to cater to individual learning needs by adapting the content based on user abilities, making it highly personalized and effective (Kechil et al., 2022). One of the major advantages of Mathigon is its visually appealing design, which captures students' attention and helps to identify misconceptions, facilitating deeper conceptual understanding (Muharram et al., 2023). Its graphics and animations break down complex mathematical ideas into manageable chunks, making the learning process engaging and immersive. A previous study by Álvarez et al. (2024) revealed that the interactive nature of Mathigon helps students visualize and understand factorization, leading to a deeper comprehension of algebraic relationships. However, Mathigon also has some limitations. While it covers many topics, certain areas may not be explored sufficiently (Ivanova, 2023). Moreover, an online platform requires a stable internet connection, which may limit access for some users (Yow & Eu, 2024). It also needs regular updates to remain effective and keep pace with the evolving demands of online education.

Mathigon, as an interactive educational platform, is specifically designed to improve students' understanding of mathematical concepts and enhance their problem-solving skills (Ferreira & Mendes, 2020). Its features include engaging content such as simulations, games, puzzles, and interactive stories that link mathematical ideas to real-life contexts. Furthermore, Mathigon employs adaptive learning technologies that personalize the learning experience for

each student, adjusting content based on their pace and style (Dahal et al., 2022; Ferreira & Mendes, 2020).

There are numerous benefits of using Mathigon in mathematics learning. It enables students to develop a different perspective on mathematics, moving beyond rote memorization to active problem-solving (Álvarez et al., 2024). Mathigon deepens students' understanding of mathematical concepts, increases learning motivation, and improves problem-solving through interactive exercises. The current study found that the Mathigon-assisted IBME approach significantly improved student engagement during Grade 8 Cartesian coordinate lessons at a junior high school in Kuala Batee. Students in the experimental group exhibited better engagement than those in the control group, consistent with research by Muir and Geiger (2016), which demonstrated the potential of the Mathigon-assisted IBME approach to substantially increase student engagement. Other studies by Archer-Kuhn et al. (2020), Attard et al. (2021), Cevikbas and Kaiser (2022), and Chen (2021) reached similar conclusions.

In this study, Mathigon facilitates interactive and personalized learning of Cartesian coordinate concepts. To support the learning process, students are provided with a specialized guide with a tutorial on using Mathigon effectively. The student worksheet contains instructions for the activities to be carried out within the Mathigon platform, all centered around Cartesian coordinate material. Mathigon is employed to help students grasp the concept of point positioning in the Cartesian coordinate plane, solve problems related to point locations, and create imaginary maps and travel routes. These activities aim to enhance students' understanding of point positions in relation to the origin and specific points within the Cartesian coordinate system.

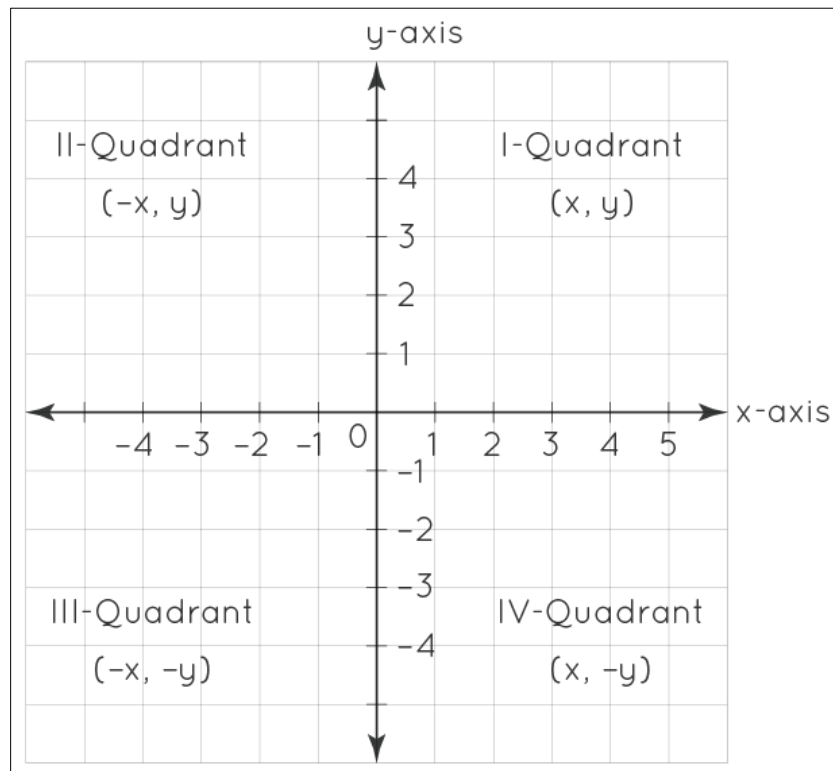
### **Cartesian coordinates**

The Cartesian coordinate system consists of two axes perpendicular to each other. These axes lie on a single plane (the  $XY$ -plane), with the horizontal axis known as the  $x$ -axis and the vertical axis known as the  $y$ -axis. The intersection point of the  $x$ -axis and the  $y$ -axis is called the origin, commonly called the zero point. Positive numbers on the  $x$ -axis start from the origin and extend to the right, while negative numbers on the  $x$ -axis start from the origin and extend to the left. Similarly, positive numbers on the  $y$ -axis start from the origin and extend upward, while negative numbers on the  $y$ -axis extend downward from the origin. The  $x$ -coordinate is often called the abscissa, and the  $y$ -coordinate is called the ordinate. Cartesian coordinate is shown in Figure 1.

In the Cartesian coordinate plane, the  $x$ -axis and  $y$ -axis divide the plane into four quadrants:

- Quadrant I: Positive  $x$ -coordinate (+) and positive  $y$ -coordinate (+)
- Quadrant II: Negative  $x$ -coordinate (-) and positive  $y$ -coordinate (+)
- Quadrant III: Negative  $x$ -coordinate (-) and negative  $y$ -coordinate (-)
- Quadrant IV: Positive  $x$ -coordinate (+) and negative  $y$ -coordinate (-)

The position of a point on the Cartesian coordinate plane is written as an ordered pair  $(x, y)$ . The  $x$ -value represents the distance of the point from the  $y$ -axis, and the  $y$ -value represents the distance from the  $x$ -axis. Determining the position of a point relative to the origin is equivalent to determining its position relative to the  $x$ -axis and  $y$ -axis, with the origin serving as the reference point. To locate a specific point, we must create  $x$  and  $y$  axes intersecting at this reference point.



**Figure 1.** Cartesian Coordinate

## Method

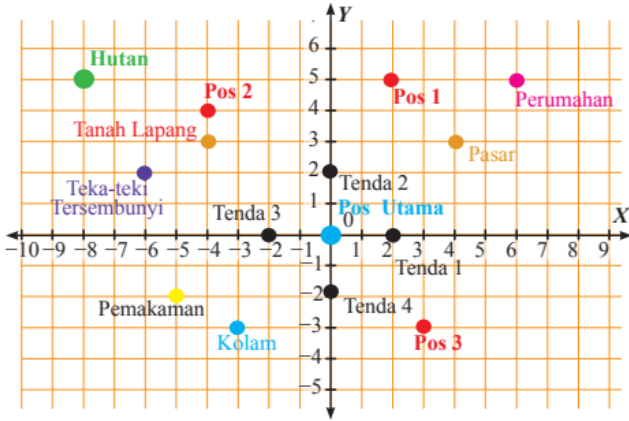
This study employed a quantitative approach using experimental methods, specifically a pre-test and post-test control group design. This design involves an initial measurement (pre-test) before the treatment, followed by the treatment itself, and concludes with a final measurement (post-test) after the treatment (Fraenkel & Wallen, 2012). The population and sample consisted of 42 Grade 8 students from a public junior high school in Kuala Batee, Aceh, Indonesia, during the 2023/2024 school year. The sample was divided into two groups: 22 students in the experimental and 20 in the control classes.

Data was collected using tests and questionnaires. The research instruments included tests assessing mathematical problem-solving abilities and a student engagement questionnaire, administered twice—before the learning session (pre-test) and after the learning session (post-test). The test comprised six problem-solving questions focused on Cartesian coordinates. The development of the test followed several stages: first, the researcher prepared the pre-test and post-test questions, including the main material, sub-material, problem indicators, cognitive levels, and problem numbers. Second, problem-solving questions were adapted from textbooks and previous studies (As'ari et al., 2017; Gustiningsi & Utari, 2020; Wirdayanti, 2023), adjusted according to a specified grid, and presented in an assessment rubric. Third, two validators reviewed the content, construction, and language test set, providing feedback for improvement. Fourth, the validity of the questions was calculated, and the test was revised based on the validators' feedback.

Finally, based on the Polya problem-solving indicators (Schoenfeld, 1987), the six questions were deemed highly valid and ready for use. Examples of the test instruments are provided below, including pre-test and post-test questions for the indicator “Students can explain the

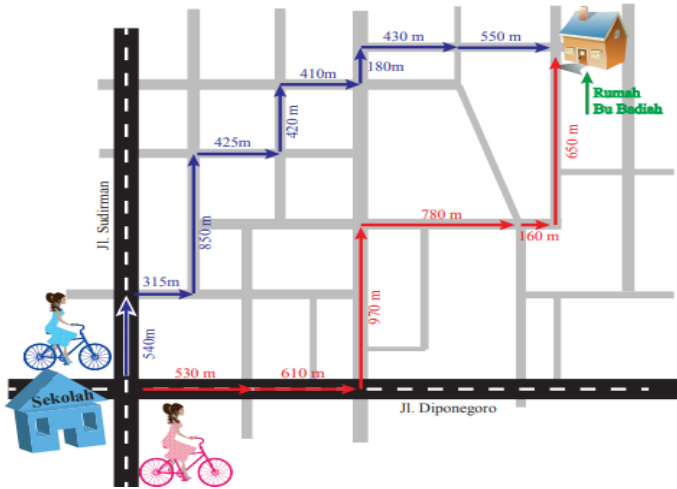
position of a point in the Cartesian plane and the position of a point relative to another point (Figure 2 and Figure 3).”

Observe the campsite map using the following Cartesian coordinates.



Based on the information above, identify the objects on the map that are equidistant from the X-axis. Provide your reasoning! Then, explain the position of the market relative to post 3!

Figure 2. Pre-test question 1



Bella and Diva want to visit their teacher, Mrs. Badiyah's house, but they are unsure of the exact address. Mrs. Badiyah only provided the information that her house is 1.78 km from Jalan Diponegoro and 2.13 km from Jalan Sudirman. Bella and Diva set off from school together using their bicycles and taking different routes. The red line shows Bella's route, and the blue line shows Diva's route, as indicated on the map. Who will reach Mrs. Badiyah's house first? Why? Can you also explain the position of Mrs. Badiyah's house relative to the school?

Figure 3. Post-test question 1

For the question indicator “Students can determine the position of a point relative to a specific point and analyze its position,” the pre-test and post-test questions are presented in Figure 4 and Figure 5.

The zookeeper assigns animal cages based on their species to assist zoo visitors. Observe the row of animal cages in the zoo below.

If the lion is coded as ML2U (Mammal Left 2 Up) and the ostrich as AR3D (Aves Right 3 Down), determine the animal cages with the codes AR1U and ML2D.

**Figure 4.** Pre-test question 2

Observe the row of parked cars in the following parking area.

C3	C2	C1		D1	D2	D3
A4	A5	A6		B6	B5	B4
A3	A2	A1		B1	B2	B3
C4	C5	C6		D6	D5	D4
Pos Jaga						

The security guard at the Watch Post creates codes for the cars' positions. For example, car number A5 is L2U1 (Left 2 Up 1), and car number D4 is R3D2 (Right 3 Down 2). Determine the car numbers for the following codes:

- a. L2D1
- b. R2U2

**Figure 5.** Post-test question 2

The student engagement questionnaire was adapted from Lam et al. (2014). Respondents marked their responses on a five-point Likert scale from 'Strongly Disagree' to 'Strongly Agree' and 'Never' to 'Always.' The questionnaire included 29 statements: 9 on emotional engagement, 8 on behavioral engagement, and 12 on cognitive engagement. The adaptation process included translating the questionnaire, consulting a language expert for revisions, and validating it with two validators for content and language. The validators' feedback was incorporated, and the questionnaire was approved without further revision.

Data analysis followed several stages to address the research questions. First, student engagement data were converted from ordinal to interval data using the Method of Successive Intervals (MSI). Next, the problem-solving abilities and student engagement data were

summarized in pre-test and post-test scores for both the experimental and control groups. Increases in mathematical problem-solving abilities and student engagement were calculated using the N-Gain formula (Hake, 2002). Normality tests using One Sample Kolmogorov-Smirnov and variance homogeneity tests using Levene's Test were conducted on the pre-test scores and N-Gain ( $\alpha = 0.05$ ). Differences in improvement between the experimental and control groups were determined using a t-test for the difference in two means ( $\alpha = 0.05$ ). The formula of the t-test is shown as follows.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$\bar{x}_1$  = Mean value of the first group

$\bar{x}_2$  = Mean value of the second group

$S$  = Standard deviation

$n_1$  = Size of the first group

$n_2$  = Size of the second group (Sudjana, 2005)

The statistical hypotheses were formulated as follows:  $H_0: \mu_E = \mu_C$  and  $H_1: \mu_E > \mu_C$ , where  $\mu_E$  is the average N-Gain of the experimental class, and  $\mu_C$  is the average N-Gain of the control class.

The flowchart of this research process is shown in Figure 6.

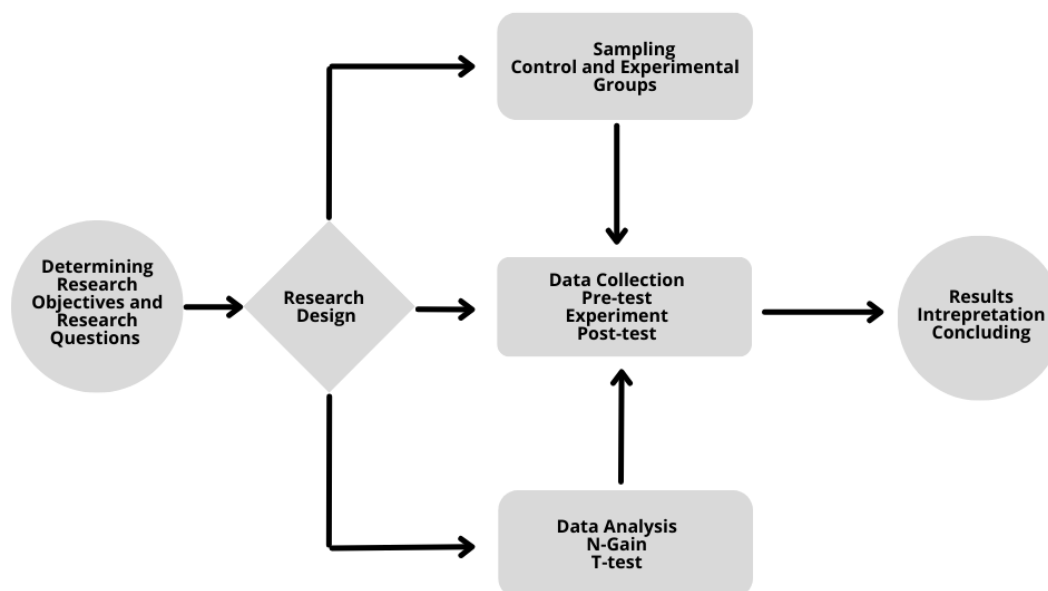


Figure 6. Research Process

The research hypotheses were:

1. "The increase in mathematical problem-solving abilities of students who receive learning using the IBME approach assisted by Mathigon is greater than that of students who receive learning using the IBME approach without Mathigon on Cartesian coordinates material."
2. "The increase in learning engagement of students who receive learning using the IBME approach assisted by Mathigon is greater than that of students who receive learning using the IBME approach without Mathigon on Cartesian coordinates material."

## Findings

This research analyzed test data on mathematical problem-solving abilities and student engagement in two classes (experimental and control classes), along with N-Gain data from the two classes. Analysis was conducted on N-Gain data to assess the increased mathematical problem-solving abilities and student engagement in both classes.

### Students' mathematical problem-solving ability

This section describes the quantitative descriptive results of the pre-test, post-test, and N-Gain data on mathematical problem-solving abilities. The data summary of mathematical problem-solving ability is shown in [Table 1](#).

**Table 1.** Mathematical problem-solving ability data summary

Student Code	Experiment Class		Gain	Ideal-Pre-test	Experiment N-Gain	Control Class		Gain	Ideal-Pre-test	Control N-Gain
	Pre-Test	Post-Test				Pre-Test	Post-Test			
S-1	24.69	88.27	63.58	75.31	0.84	37.04	88.27	51.23	62.96	0.81
S-2	50.62	91.98	41.36	49.38	0.84	22.84	91.98	69.14	77.16	0.90
S-3	40.12	80.86	40.74	59.88	0.68	33.95	74.69	40.74	66.05	0.62
S-4	33.95	82.10	48.15	66.05	0.73	59.26	82.10	22.84	40.74	0.56
S-5	21.60	83.33	61.73	78.40	0.79	54.94	81.48	26.54	45.06	0.59
S-6	3.09	67.90	64.81	96.91	0.67	27.16	71.60	44.44	72.84	0.61
S-7	53.70	85.80	32.10	46.30	0.69	40.74	77.16	36.42	59.26	0.61
S-8	30.86	87.04	56.17	69.14	0.81	27.16	87.04	59.88	72.84	0.82
S-9	6.17	77.16	70.99	93.83	0.76	19.14	54.32	35.19	80.86	0.44
S-10	21.60	80.86	59.26	78.40	0.76	45.06	80.86	35.80	54.94	0.65
S-11	56.79	77.78	20.99	43.21	0.49	37.65	72.22	34.57	62.35	0.55
S-12	47.53	72.22	24.69	52.47	0.47	38.89	50.00	11.11	61.11	0.18
S-13	40.12	82.10	41.98	59.88	0.70	48.77	79.01	30.25	51.23	0.59
S-14	38.27	80.86	42.59	61.73	0.69	30.25	66.05	35.80	69.75	0.51
S-15	18.52	69.14	50.62	81.48	0.62	28.40	56.79	28.40	71.60	0.40
S-16	52.47	80.25	27.78	47.53	0.58	30.86	69.75	38.89	69.14	0.56
S-17	46.30	74.07	27.78	53.70	0.52	46.91	74.07	27.16	53.09	0.51
S-18	33.95	82.72	48.77	66.05	0.74	22.22	45.06	22.84	77.78	0.29
S-19	55.56	79.63	24.07	44.44	0.54	29.01	74.07	45.06	70.99	0.63
S-20	52.47	91.36	38.89	47.53	0.82	44.44	82.72	38.27	55.56	0.69
S-21	33.95	87.04	53.09	66.05	0.80					
S-22	63.58	75.31	11.73	24.69	0.48					

[Table 2](#) shows that the experimental class's average pre-test score for mathematical problem-solving abilities was 37.54, while the control class scored 36.23. After the intervention, the average post-test score in the experimental class was 80.81, compared to 72.96 in the control class. This indicates an improvement in problem-solving abilities in both classes.

**Table 2.** Mathematical problem-solving ability data

Data	Experiment		Control	
	N	$\bar{X}$	N	$\bar{X}$
Pre-test	22	37.54	20	36.23
Post-test	22	80.81	20	72.96

Table 3 shows that the N-Gain value for the experimental class was 15.01, while for the control class, it was 11.54. Both values fall within the medium criteria, but the experimental class showed a greater increase, with a difference of 3.47.

**Table 3.** N-Gain data for mathematical problem-solving ability

Data	Experiment	Control
N	22	20
N-Gain	15.01	11.54
Average	0.68	0.58
Criteria	Medium	Medium

Ideal score = 100

Table 4 shows that the pre-test scores for mathematical problem-solving abilities in both the experimental and control classes were not significant ( $p > 0.05$ ), indicating that the data were normally distributed.

**Table 4.** Normality of pre-test scores for mathematical problem-solving ability

Data	Kolmogorov Smirnov			Shapiro-Wilk			Decision
	Statistic	df	Sig.	Statistic	df	Sig.	
Pre-test_Experiment	.128	20	.200	.929	20	.149	H <sub>0</sub> accepted
Pre-test_Control	.136	20	.200	.966	20	.660	H <sub>0</sub> accepted

Table 5 indicates that the pre-test scores for mathematical problem-solving abilities were not significant ( $p = 0.089$ ), showing that the data were homogeneous.

**Table 5.** Homogeneity of pre-test score variance for mathematical problem-solving ability

Lavene Statistic	df1	df2	Sig.	Decision
3.033	1	40	.089	H <sub>0</sub> accepted

Table 6 shows that the N-Gain scores for both the experimental and control classes were not significant ( $p > 0.05$ ), indicating that the data were normally distributed.

**Table 6.** Normality of N-Gain scores for mathematical problem-solving ability

Data	Kolmogorov Smirnov			Shapiro-Wilk			Decision
	Statistic	df	Sig.	Statistic	df	Sig.	
Pre-test_Experiment	.142	20	.200	.933	20	.176	H <sub>0</sub> accepted
Pre-test_Control	.148	20	.200	.956	20	.467	H <sub>0</sub> accepted

Table 7 shows that the N-Gain scores for mathematical problem-solving abilities were not significant ( $p > 0.05$ ), indicating homogeneity of variance.

**Table 7.** Homogeneity of N-Gain score variance for mathematical problem-solving ability

Lavene Statistic	df1	df2	Sig.	Decision
.418	1	40	.521	H <sub>0</sub> accepted

After confirming the normality and homogeneity of the N-Gain data, the average difference test using the t-test was performed. The results are displayed in [Table 8](#).

**Table 8.** Average difference of N-Gain scores for mathematical problem-solving ability

	Levene's Test for Equality of Variances				t-test for Equality of Means				
	F	Sig.	t	df	Sig.(2-tailed)	Mean Difference	Std.Error Difference	95% Confidence Interval of the Difference	
								Lower	Upper
Equal Variances assumed	.418	.521	2.362	40	.023	.10673	.04519	.01539	.19807
Equal Variances not assumed			2.325	34.292	.026	.10673	.04590	.01349	.19997

[Table 8](#) shows a significant difference in the N-Gain scores between the control and experimental classes (M = 0.10673, SE = 0.04519, t(40) = 2.362, p = 0.023). Therefore, it can be concluded that the increase in students' mathematical problem-solving abilities in Cartesian coordinates was greater in the experimental class taught using the IBME approach assisted by Mathigon compared to the control class using the IBME approach without Mathigon assistance.

### Students learning engagement

This section presents the quantitative descriptive results of the pre-test, post-test, and N-Gain data on learning engagement. The data has been transformed from ordinal to interval form. The data summary of students' learning engagement is shown in [Table 9](#).

**Table 9.** Students learning engagement data summary

Student Code	Experiment Class		Gain	Ideal Pre-test	Experiment N-Gain	Control Class		Gain	Ideal-Pre-test	Control N-Gain Pre-Test
	Pre-Test	Post-Test				Pre-Test	Post-Test			
S-1	24.69	88.27	63.58	75.31	0.84	37.04	88.27	51.23	62.96	0.81
S-2	50.62	91.98	41.36	49.38	0.84	22.84	91.98	69.14	77.16	0.90
S-3	40.12	80.86	40.74	59.88	0.68	33.95	74.69	40.74	66.05	0.62
S-4	33.95	82.10	48.15	66.05	0.73	59.26	82.10	22.84	40.74	0.56
S-5	21.60	83.33	61.73	78.40	0.79	54.94	81.48	26.54	45.06	0.59
S-6	3.09	67.90	64.81	96.91	0.67	27.16	71.60	44.44	72.84	0.61
S-7	53.70	85.80	32.10	46.30	0.69	40.74	77.16	36.42	59.26	0.61
S-8	30.86	87.04	56.17	69.14	0.81	27.16	87.04	59.88	72.84	0.82
S-9	6.17	77.16	70.99	93.83	0.76	19.14	54.32	35.19	80.86	0.44
S-10	21.60	80.86	59.26	78.40	0.76	45.06	80.86	35.80	54.94	0.65
S-11	56.79	77.78	20.99	43.21	0.49	37.65	72.22	34.57	62.35	0.55
S-12	47.53	72.22	24.69	52.47	0.47	38.89	50.00	11.11	61.11	0.18

Student Code	Experiment Class		Gain	Ideal Pre-test	Experiment N-Gain	Control Class		Gain	Ideal-Pre-test	Control N-Gain Pre-Test
	Pre-Test	Post-Test				Pre-Test	Post-Test			
S-13	40.12	82.10	41.98	59.88	0.70	48.77	79.01	30.25	51.23	0.59
S-14	38.27	80.86	42.59	61.73	0.69	30.25	66.05	35.80	69.75	0.51
S-15	18.52	69.14	50.62	81.48	0.62	28.40	56.79	28.40	71.60	0.40
S-16	52.47	80.25	27.78	47.53	0.58	30.86	69.75	38.89	69.14	0.56
S-17	46.30	74.07	27.78	53.70	0.52	46.91	74.07	27.16	53.09	0.51
S-18	33.95	82.72	48.77	66.05	0.74	22.22	45.06	22.84	77.78	0.29
S-19	55.56	79.63	24.07	44.44	0.54	29.01	74.07	45.06	70.99	0.63
S-20	52.47	91.36	38.89	47.53	0.82	44.44	82.72	38.27	55.56	0.69
S-21	33.95	87.04	53.09	66.05	0.80					
S-22	63.58	75.31	11.73	24.69	0.48					

As shown in Table 10, the average pre-test score for student engagement in the experimental class was 75.46, compared to 78.08 in the control class. The average post-test score was 83.88 in the experimental class and 81.90 in the control class. This indicates an increase in student engagement in both classes.

**Table 10.** Learning engagement data

Data	Experiment		Control	
	N	$\bar{X}$	N	$\bar{X}$
Pre-test	22	75.46	20	78.08
Post-test	22	83.88	20	81.90

Table 11 shows that the total N-Gain for student engagement in the experimental class was 7.54, compared to 3.22 in the control class, resulting in a difference of 4.32. This demonstrates a greater improvement in the experimental class.

**Table 11.** N-Gain data for learning engagement

Data	Experiment	Control
N	22	20
N-Gain	7.54	3.22
Average	0.34	0.16
Criteria	Medium	Medium

Table 12 indicates that the pre-test scores for student engagement in the experimental and control classes were insignificant ( $p > 0.05$ ), meaning  $H_0$  is accepted, and the data are normally distributed.

**Table 12.** Normality of pre-test scores for learning engagement

Data	Kolmogorov Smirnov			Shapiro-Wilk			Decision
	Statistic	df	Sig.	Statistic	df	Sig.	
Pre-test_Experiment	.108	20	.200	.968	20	.723	$H_0$ accepted
Pre-test_Control	.096	20	.200	.961	20	.568	$H_0$ accepted

Table 13 shows that the pre-test scores for engagement in the experimental and control classes were insignificant ( $p > 0.05$ ), indicating homogeneity of variance.

**Table 13.** Homogeneity of pre-test score variance for learning engagement

Lavene Statistic	df1	df2	Sig.	Decision
.438	1	40	.512	H <sub>0</sub> accepted

Table 14 shows that the N-Gain scores for student engagement in the experimental and control classes were insignificant ( $p > 0.05$ ), meaning H<sub>0</sub> is accepted, and the data are normally distributed.

**Table 14.** Normality of N-Gain scores for learning engagement

Data	Kolmogorov Smirnov			Shapiro-Wilk			Decision
	Statistic	df	Sig.	Statistic	df	Sig.	
Pre-test_Experiment	.153	20	.200	.917	20	.086	H <sub>0</sub> accepted
Pre-test_Control	.113	20	.200	.973	20	.812	H <sub>0</sub> accepted

Table 15 indicates that the N-Gain engagement scores for the experimental and control classes were insignificant ( $p > 0.05$ ), indicating homogeneity of variance.

**Table 15.** Homogeneity of N-Gain score variance for learning engagement

Lavene Statistic	df1	df2	Sig.	Decision
4.067	1	40	.050	H <sub>0</sub> accepted

Table 16 shows a significant difference in learning engagement between the control and experimental classes ( $M = 0.18123$ ,  $SE = 0.05043$ ,  $t(40) = 3.593$ ,  $p = 0.000$ ). Therefore, it can be concluded that the IBME approach assisted by Mathigon significantly improves student learning engagement compared to the IBME approach without Mathigon's assistance.

**Table 16.** Average difference of N-Gain scores for learning engagement

	Levene's Test for Equality of Variances				t-test for Equality of Means				
	F	Sig.	t	df	Sig.(2-tailed)	Mean Difference	Std.Error Difference	95% Confidence Interval of the Difference	
								Lower	Upper
Equal Variances assumed	4.067	.050	3.593	40	.001	.18123	.05043	.07930	.28316
Equal Variances not assumed			3.517	31.345	.001	.18123	.05152	.07619	.28626

## Discussion

This research aims to assess the improvement in problem-solving abilities and student engagement using the Mathigon-assisted Inquiry-Based Mathematics Education (IBME) approach in teaching Cartesian coordinates for junior high school students. The study was conducted at a junior high school in Kuala Batee during the 2023/2024 academic year. It involved four experimental and control group meetings, focusing on Cartesian coordinates for

Grade 8 students. Data analysis, which included both descriptive and inferential statistical tests, showed that using the Mathigon-assisted IBME approach led to a greater increase in students' mathematical problem-solving abilities and engagement compared to traditional methods. In this study, Mathigon interactive tools make mathematical concepts, like Cartesian coordinates, more tangible and engaging for students. Features such as visual aids, interactive exercises, and instant feedback allow students to grasp complex ideas more effectively and apply them confidently, enhancing their problem-solving skills. The Inquiry-Based Mathematics Education (IBME) approach complements this by promoting active participation, encouraging students to ask questions and explore solutions independently. Statistical analysis confirmed a significant improvement in both areas when using this approach.

This research builds on previous studies that emphasize the role of inquiry-based learning in enhancing students' problem-solving abilities (Schallert et al., 2022). Inquiry-based learning encourages students to think creatively and systematically while solving problems by helping them develop strategies, understand problems thoroughly, and plan and execute solutions (Abdurrahman et al., 2021). Studies also indicate that inquiry-based learning, supported by metacognitive strategies, improves problem-solving skills and engagement (Divrik et al., 2020). Similarly, research shows that students exposed to inquiry-based methods, such as advanced organizer approaches, exhibit better focus, planning, and execution when tackling mathematical problems (Gunawan et al., 2020).

In this study, students taught using the Mathigon-assisted IBME approach showed significant improvements in all four indicators of problem-solving abilities. For example, 80% of students in the experimental group could devise effective plans for solving Cartesian coordinate problems, compared to only 55% in the control group. Additionally, more students in the experimental group successfully reviewed and refined their solutions, an essential part of the problem-solving process (Schallert et al., 2022).

The research also revealed that the Mathigon-assisted IBME approach significantly enhanced student engagement in the experimental group compared to the control group. In this study, Mathigon's interactive tools and adaptive learning paths made the material more accessible by allowing students to learn at their own pace, creating a personalized experience that kept them engaged. The students feel more comfortable with the content, increasing their willingness to actively participate. Combined with the IBME approach, Mathigon's visually engaging and interactive features stimulated students' curiosity, prompting them to explore Cartesian coordinates independently and discover solutions through hands-on learning. This sense of discovery led to deeper engagement, as students were motivated to solve problems themselves. Engagement was measured through behavioral, cognitive, and emotional indicators. For instance, 90% of students in the experimental group actively participated in discussions about problem-solving strategies, compared to only 60% in the control group. Emotional engagement was notably higher among Mathigon students, who expressed greater interest and enthusiasm in learning Cartesian coordinates through interactive simulations and puzzles. This finding aligns with research by Ryan and Deci (2000), which emphasizes the role of novel and challenging activities in boosting engagement. Mathigon's features, such as interactive animations, puzzles, and adaptive learning technologies, enhanced students' cognitive engagement. Mathigon students were more inclined to explore multiple problem-solving strategies, engage deeply with mathematical concepts, and retain knowledge of Cartesian coordinates more effectively, as shown by their improved post-test results.

The novelty and engaging nature of the Mathigon-assisted IBME approach likely contributed to the increased student interest and enthusiasm observed in the study, consistent with research indicating that new, challenging, or aesthetically valuable learning activities enhance engagement (Ryan & Deci, 2000). High levels of engagement maximize the learning process, leading to better mathematical understanding and outcomes (Anggari and Rufiana, 2020). Implementing the IBME approach to enhance engagement involves presenting interesting and relevant mathematical problems, promoting active exploration, encouraging discussion and collaboration, and allowing students to present their findings (Schallert et al., 2022). This approach also improves motivation, critical and creative thinking, and mathematical communication skills (Dorier & Maass, 2020). Increased engagement through Mathigon can be measured by participation in learning activities, interest and enthusiasm, understanding of mathematical concepts, achievement of learning goals, and involvement in discussions about the content (Dahal et al., 2022).

## Conclusion

Based on the research and discussion presented, it can be concluded that the IBME approach assisted by Mathigon significantly enhances mathematical problem-solving abilities and student learning engagement compared to the IBME approach without Mathigon's assistance. Given these conclusions, the IBME approach with Mathigon is recommended for teachers aiming to improve students' problem-solving abilities and engagement. Effective implementation of the IBME approach requires careful planning and appropriate time management.

This research focused solely on Cartesian coordinates for Grade 8. Future research should extend to other mathematical topics and include larger sample sizes to generalize findings at the junior high school level. Additionally, future studies should employ diverse data collection techniques for student engagement, such as classroom observations and interviews, to complement questionnaire data.

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