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Improving students' mathematical reasoning and self-efficacy through Missouri mathematics project and problem-solving

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Abstrak: Artikel ini bertujuan untuk mendeskripsikan keefektifan pembelajaran *missouri mathematics project* dengan pendekatan *problem solving* (MMP-PS) terhadap kemampuan penalaran matematika dan *self-efficacy*. Penelitian kuasi eksperimen dilaksanakan dengan melibatkan 132 siswa SMP (kisaran umur 14-15 tahun) sebagai populasi. Dipilih secara acak dua kelas yang terdiri dari masing-masing 25 siswa sebagai kelas eksperimen dan kelas kontrol. Kelas eksperimen diajar dengan menggunakan pembelajaran *Missouri mathematics project* dengan pendekatan *problem solving* dan kelas kontrol diajar dengan pembelajaran *missouri mathematics project* (MMP). Data penelitian dianalisis melalui uji satu sampel untuk mengukur keefektifan MMP-PS dan MMP terhadap kemampuan penalaran matematika dan *self-efficacy*. Perbedaan kemampuan penalaran matematika dan *self-efficacy* siswa sebelum dan sesudah pembelajaran dianalisis menggunakan uji Manova. Perbandingan keunggulan antara MMP-PS dan MMP menggunakan uji *t-Benferroni*. Hasil penelitian menunjukkan bahwa MMP-PS efektif ditinjau dari kemampuan penalaran matematika dan *self-efficacy*. Sementara itu, pembelajaran MMP efektif ditinjau kemampuan penalaran matematika. Pembelajaran MMP-PS lebih unggul dari pembelajaran MMP ditinjau dari *self-efficacy* siswa. Jadi, MMP-PS bisa digunakan dalam pembelajaran matematika untuk mendukung kemampuan penalaran matematika dan *self-efficacy*.

Kata kunci: *Missouri mathematics project, Penalaran matematika, Self-efficacy, Pemecahan masalah*

Abstract: This article aims to elucidate the effectiveness of Missouri mathematics project-problem solving learning (MMP-PS) toward mathematical reasoning ability and self-efficacy. We conducted quasi-experiment research which involve 132 lower secondary school students (age 14-15 years old) as the population. Of the population, 25 students respectively in two classes were selected randomly as experiment class and control class. The experiment class was taught using MMP-PS, and a control class was taught using MMP (Missouri mathematics project learning). Data were analyzed using one sample t-test to examine the effectiveness of MMP-PS and MMP toward mathematics reasoning ability and self-efficacy. The difference of students' mathematical reasoning and self-efficacy before and after the treatment was analyzed using Manova. The comparison of the effectiveness of MMP-PS and MMP were analyzed through a t-Benferroni test. This article shows that MMP-PS is effective on mathematical reasoning ability and self-efficacy. Meanwhile, MMP is effective on mathematical reasoning ability. The MMP-PS was better than MMP on students' self-efficacy. Thus, MMP-PS could be utilized in mathematics learning to support students' mathematical reasoning ability and self-efficacy.

Keywords: *Missouri mathematics project, Problem-solving, Mathematical reasoning, self-efficacy*

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A. Introduction

One of the most essential aspect to developed on students in mathematics education is mathematical reasoning ability. The reasoning is important in learning mathematics because it helps students to construct their knowledge and support their academic development (Ongcoy, 2016). The reasoning is an activity that involves finding patterns, making predictions, evaluating conjectures, and constructing valid evidence or mathematical arguments (NCTM, 2000). Mathematical concepts can be understood through reasoning, and otherwise, reasoning can be trained and developed through mathematics. Therefore, mathematics and reasoning are two things that cannot be separated because students can learn mathematics well by reasoning (Maarif, 2016).

The importance of mathematical reasoning ability in learning mathematics is not as smooth as the conditions of students when they learn mathematics. Data of Trends in the International Mathematics and Science Study (TIMSS) in 2011 showed that average mathematics score of Indonesian students was 386 which was below the scale of the center point of 500. Besides, the ability of Indonesian students to knowing, applying, and reasoning mathematics concept was the lowest amongst other countries in the same context (Mullis, Martin, Foy, & Arora, 2012).

The reasoning is often interpreted as the process of drawing a conclusion based on evidence (NCTM, 2009). Conner, Singletary, Smith, Wagner, and Fransisco (2014) reveal that reasoning is the process of drawing a conclusion or cognitive process to arrive at a conclusion based on information that has been known. Santrock (2011) defines reasoning as logical thinking which involves induction and deduction to conclude. Thus, it should be realized that not all thinking activities are reasoning activities. Reasoning activities emphasize higher-order thinking processes that are not only limited to answering what question, but also answering why and finally come to a conclusion.

In mathematics, reasoning can include some forms such as giving explanations, doing justifying, conducting searches, predicting, or giving conjecture (NCTM, 2009; Lim, Kim, Cordero, Buendia, & Kasmer, 2015). Brodie (2010) states that *"…mathematical reasoning is essentially about the development, justification and use of mathematical generalizations (p.9)."* Referring to Lim et al. (2015) and Brodie (2010), justification or compiling valid evidence to prove the mathematical argument is the important thing of mathematical reasoning. NCTM (2000) explains that students in sixth until eighth grade must have mathematical reasoning ability that is realized through activities such as *"...examine pattern and structures to detect regularities, formulate generalizations and conjectures about observed regularities, evaluate conjectures, construct and evaluate mathematical arguments…(p.262)."* Drawing from the arguments about mathematical reasoning, mathematical reasoning ability is measured in this study through three aspects, namely (a) the ability to find a pattern of a mathematical phenomenon, (b) the ability to make conjectures, and (c) the ability to give a reason for the solution.

Beside cognitive aspect, affective aspect also has a vital role for the student in mathematics learning. One of affective aspect is self-efficacy. Malpass, O'neil, and Hocevar (2010) reveal that self-efficacy affects the achievement student in learning. Self-efficacy is the belief in a person's ability to himself so that he is motivated to achieve planned success. On the other hand, many researchers (e.g., Schunk & Meece, 2006; McCoach, Gable, & Madura, 2013; Sharma & Nasa, 2014; Greene, 2018) argue that self-efficacy is domain specific and perception of students toward their capability to study or success in accomplish a task.

When linking to mathematics, Betz and Hackett (1983) reveal that self-efficacy is the assessment of students on their ability to solve mathematics problems, complete mathematics tasks, or success related to mathematics. Kitsantas, Cheema, and Ware (2011) found that students who have low mathematics score tend to have low self-efficacy and spend more time solving the problem. This fact shows that between self-efficacy and mathematics achievement of students correlate (Phan, 2012). Phan (2012) strengthen that self-efficacy affected achievement of student in learning mathematics. Therefore, self-efficacy is an affective aspect that students must improve in learning mathematics.

There are four sources of self-efficacy that can be used as guidelines to develop self-efficacy of students, namely: performance accomplishments, vicarious experience, verbal persuasion, and emotional arousal (Bandura, 2009). Performance accomplishments mean that individual success can increase self-efficacy or otherwise. Vicarious experience relates to the experience of others can also act as an individual source of information. The individual does not only depend on his personal experience of success as a source of self-efficacy but by looking at the success of others who can foster individual trust that he can do it. Verbal persuasion is related to the responses and judgments of parents, teachers or peers. Positive responses or constructive comments can encourage student's self-efficacy. Emotional arousal appears to be a very influential source of individual self-efficacy. The emotional condition in question is a feeling of stress, anxiety, fatigue and mood (Lau, Kitsantas, Miller, & Rodgers, 2018). The four aspects of developing self-efficacy could be capitalized in mathematics learning.

Considering the importance of mathematical reasoning ability and self-efficacy in learning mathematics, the teacher has to develop innovation in teaching. The innovation can be choosing an approach in learning to improve mathematical reasoning ability and self-efficacy. One of the approaches is applying Missouri mathematics project embedded with problem-solving. Good and Grouws (1979) explain that Missouri mathematics project utilizes the exercises to improve student learning achievement. Furthermore, Jannah, Triyanto, and Ekana (2013) account that Missouri mathematics project is a learning model designed by combining independence work and collaborative work among students. Setyawan, Budiyono, and Slamet (2017) explicate that Missouri mathematics project is a learning model that aims to develop the concept of mathematics.

Good and Grouws (1979) explained that the steps of Missouri mathematics project learning consisted of daily review, development, seatwork, homework assignment, and special review. Meanwhile, Reynold and Muijs (1999) added that Missouri mathematics project learning follows four steps namely daily review, development, individual work, homework assignment. The other sources (e.g., Krismanto, 2003, Handayani, Januar, & Purwanto, 2018) list Missouri mathematics project as a review, development, cooperative work, seatwork or individual work, and giving homework.

Rivai and Surya (2017) found that there is an increase in the indicators of reasoning ability in students who were taught using Missouri mathematics project learning. Relevant studies (e.g., Handayani, Januar, & Purwanto, 2018; Widyawati, 2017) showed that MMP learning is effective to be applied to support student's learning achievement and problem-solving ability. Ulya and Hidayah (2016) showed that Missouri mathematics project could improve student's self-efficacy. In this study, during the learning process using MMP students solve math problems which was given in projects and solve it through activities such as explaining what is known and asked, making plans, and drawing a sketch. The activity aims to make students actively involved in learning so that they have sufficient knowledge. The implication is that when students have sufficient knowledge about the topics (concepts of mathematics), they will feel confident in their ability to solve further problems.

To distinguish this study to other studies (e.g., Handayani, Januar, & Purwanto, 2018; Widyawati, 2017), we applied MMP learning which was combined with the problem-solving approach (MMP-PS) toward mathematical reasoning ability and self-efficacy. Problem-solving is an approach that aims to help students learn how to solve the problem through hands-on learning experience (Jacobsen, Eggen, & Kauchak, 2009). It is also defined as learning that uses the problems of daily activities and problem situations (real problems) that are simulated as a context for learning mathematics (Schroeder & Lester, 1989; Sajadi, Amiripour, & Malkhalifeh, 2013). The problems used can also be a non-routine problem. Non-routine problem is a problem where there is no direct access or a sure way to solve it (Wijaya, 2012). Thus, MMP-PS learning in this study utilizes non-routine and contextual problem.

Learning through problem-solving is not using mathematics to solve the problem, but learning mathematics through solving the problem so the student can get new knowledge (Johnson & Schmidt, 2006). Problem-solving has four steps, i.e., understand the problem, devise a plan, carry out the plan, and looking back (Polya, 1988). Natsusaka (2007) also applied four steps in problem-solving; identify a problem, develop a solution, discussion through the presentation of a result, and drawing a conclusion. Melianingsih and Sugiman (2015) found that problem-solving learning had a positive impact on students' mathematical reasoning ability. It can be seen from the results of statistical tests which showed that there was an increase in the average value of students and the maximum score obtained by students on the test of mathematical reasoning ability after being taught using problem-solving learning. Another study by Sulak (2010) showed that problem-solving was significantly successful in improving students' ability to make drawings, diagrams, or tables, make mathematical statements, identify patterns, make the list of strategies, use logical reasoning, and check strategic. Meanwhile, Martyanti (2016) showed that learning with a problem-solving approach was effective in increasing students' self-confidence. The results of this study indicate that learning with a problem-solving approach will also have a positive effect on self-efficacy because self-confidence and self-efficacy are related.

In this study, we sought to answer and explore three questions, i.e., Is the MMP-PS learning effective toward mathematical reasoning ability and self-efficacy? Is the MMP learning effective toward mathematical reasoning ability and self-efficacy? Which one is better toward mathematical reasoning ability and self-efficacy? The three questions are important to be answered for helping mathematics teachers to choose an alternative learning approach which support cognitive (reasoning) and affective aspect (self-efficacy) as well.

B. Methods

The study we conducted was a quasi-experiment. The population was all eighth-grade classes consisting of 132 students (age 14-15 years). We selected randomly two classes as experiment class and control class where each class consisted of 25 students. In experiment class, students were taught with MMP-PS and control class was taught using MMP (Missouri mathematics project learning). The learning steps of MMP-PS are synthesized from existing literature on MMP and problem-solving (e.g., Schroeder & Lester, 1989; Sajadi, Amiripour, & Malkhalifeh, 2013). Meanwhile, the steps of MMP learning (Table 1) are also adapted from prior works (e.g., Reynold & Muijs, 1999; Krismanto, 2003; Handayani, Januar, & Purwanto, 2018).

Table 1. Learning steps of MMP-PS and MMP

Steps	MMP-PS	MMP
1	*Review* a. Review the previous lesson b. Ask some computation exercise c. Checking homework at the previous meeting d. Inviting to observe objects around them that relate to the concept	*Review* Similar to MMP-PS
2	*Develop material (mathematical concepts)* Each group gets a worksheet and then solve the problem by steps: a. Understanding the problem b. Devising a plan c. Carrying out the plan d. Looking back Solving several problems cooperatively	*Subject development/control exercise* a. Briefly focus on prerequisite concepts b. Focus on introducing new concepts and improving student's understanding by explanation, giving questions, etc. c. Using control exercise
3	*Seatwork (Individual work)* The teacher gives assignments to students in the form of problems that are the same as those on the worksheet or variations of mathematics problem that prepared by the teacher. The problem given by the teacher in this step are carried out independently by the student	*Seatwork (Individual work*) Similar to MMP-PS
4	*Closing* a. Giving summarize regular basic of the mathematics concepts b. Review one or two problems at the end	*Closing* Similar to MMP-PS

This study started with administering pre-test for two classes (experiment class and control class) to see whether or not students in both classes have similar prior knowledge. It is done to ensure that prior knowledge of students in both classes is same so that the treatment that has been designed can be done and the final results can be determined which learning is effective or ineffective. The lessons were carried out eight meetings to adjust with basic competence and time allocation. After the learning was done, the students were given post-test to examine the effect of the learning that had been applied.

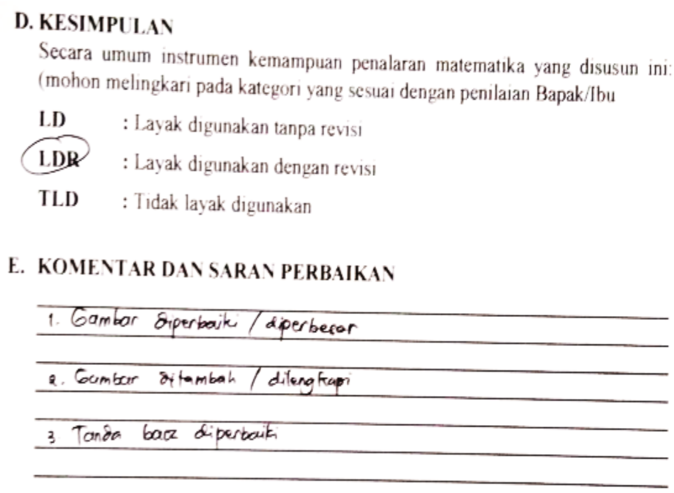
The instrument used to collect the data consisted of mathematical reasoning ability test on circle topic and questionnaire of students' self-efficacy in learning mathematics. There were four problems in mathematical reasoning ability test to measure each aspect of mathematical reasoning ability in this study: (a) the ability to find a pattern of a mathematical phenomenon; (b) the ability to make conjectures; (c) the ability to give a reason for the solution. Researchers independently developed the test to measure the students' mathematical reasoning ability. The problems developed also refer to the characteristics of the circle topic in the eighth-grade class. Furthermore, the questions that have been made were then discussed further with the second author and validated by two expert judgments (content validity). Content validity in this study consisted of face validity and logical validity. The result of content validity by two expert judgments were proper to use.

There were 30 items to measure the self-efficacy of students. The 30 statements were divided into three aspects of self-efficacy namely level (magnitude), strength, and generality. The level is the belief in ability related to the difficulty degree of the problem. Strength is a belief in abilities related to resilience and tenacity of students to solve mathematics problems. Generality is related to student's belief to be consistent in various situations. The questionnaire was developed referring to an existing instrument or theories (e.g., Bandura, 2009; Wasida, 2016; Wahyudi, 2017). The indicators of self-efficacy in each aspect can be seen in Table 2.

Table 2. Excerpts of a self-efficacy questionnaire

Aspect	Indicators	Samples of questionnaire item
Level	Confidence in the chosen strategy	I am confident in my way to solve mathematics problems without influence from friends
	Confidence in ability to solve mathematics problem with the different level of difficulty	I am sure that I can resolve difficulties in math assignments
Strength	Belief with the effort which has done	I doubt that I can improve math achievement even though I have studied well
	Resilience and tenacity of the student in solving a mathematics problem	I study various learning resources when I have difficulty working on a math problem
Generality	To believe can solve some different mathematics problems	I doubt to solve the mathematics problems that are presented in the word problem
	Confidence is consistent in every situation	I dare to express my opinion when studying mathematics in groups

The validity of instrument in this study consisted of content validity obtained from expert judgment and construct validity. Results of content validity showed that all instruments were proper for use. The result of content validity can be seen in Figure 1 and 2.


D. KESIMPULAN

Secara umum instrumen kemampuan penalaran matematika yang disusun ini:
(mohon melingkari pada kategori yang sesuai dengan penilaian Bapak/Ibu

LD : Layak digunakan tanpa revisi

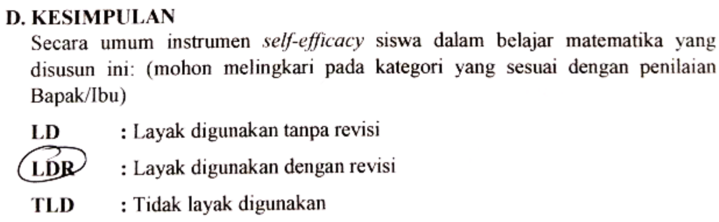
LDR : Layak digunakan dengan revisi

TLD : Tidak layak digunakan

E. KOMENTAR DAN SARAN PERBAIKAN

1. Gambar diperbaiki / diperbesar
2. Gambar ditambah / dilengkapi
3. Tanda baca diperbaiki

Figure 1. Content validity result of mathematical reasoning ability test


D. KESIMPULAN
Secara umum instrumen *self-efficacy* siswa dalam belajar matematika yang disusun ini: (mohon melingkari pada kategori yang sesuai dengan penilaian Bapak/Ibu)

LD : Layak digunakan tanpa revisi
LDR : Layak digunakan dengan revisi
TLD : Tidak layak digunakan

Figure 2. Content validity result of a self-efficacy questionnaire

Construct validity is obtained from factor analysis. The instrument item is said to be valid if the value of *Kaiser-Mayer-Olkin Measure of Sampling Adequac*y (KMO) > 0,5. On construct validity, the analysis test using KMO value is 0,608. It means that factor analysis can be continued and showed that all statement items in the self-efficacy questionnaire could be used.

Reliability of the instruments was tested in other classes. Reliability of all instrument in this study used *Alpha Cronbach* formula (Ebel & Frisbie, 1986) as follows

$$r = \frac{k}{k-1}\left(1 - \frac{{s_i}^2}{{s_t}^2}\right)$$

r = instrument reliability coefficient
k = number of items
${s_i}^2$ = variance of student score on an item
${s_t}^2$ = variance of total score

Reliability of mathematical reasoning ability is 0,756 and self-efficacy is 0,892. The reliability of the instrument was more than 0,65 which mean that the instrument is reliable to use in this study.

Data analysis in this study consisted of descriptive analysis and inferential analysis. Descriptive analysis was done by calculating descriptive statistics such as mean, standard deviation, minimum value and maximum value that might be obtained by students. However, especially for the data of self-efficacy, the total score was converted from quantitative data into qualitative data by following criteria.

Table 3. Data conversion from quantitative to qualitative

Interval	Criteria
X > $(\bar{X} + 1,5s)$	Very High
$(\bar{X} + 0,5s)$ < X ≤ $(\bar{X} + 1,5s)$	High
$(\bar{X} - 0,5s)$ < X ≤ $(\bar{X} + 0,5s)$	Moderate
$(\bar{X} - 1,5s)$ < X ≤ $(\bar{X} - 0,5s)$	Low
X ≤ $(\bar{X} - 1,5s)$	Very Low

NB:
$\bar{X}$ (mean ideal) = $\frac{1}{2}$ (maximum score + minimum score)
s (standard deviation ideal) = $\frac{1}{6}$ (maximum score - minimum score)

Minimal and maximum self-efficacy scores that may be obtained by students when filling in all statements are 30 and 150 respectively so that it is obtained $\bar{X} = 90$ and $s = 20$. Based on the criteria for converting quantitative to qualitative data, the conversion of data on self-efficacy of students is as follows.

Table 4. Qualitative criteria of self-efficacy score

Interval	Criteria
X > 120	Very High
100 < X ≤ 120	High
80 < X ≤ 100	Moderate
60 < X ≤ 80	Low
X ≤ 60	Very Low

Meanwhile, the inferential analysis, the data analysis technique consisted of three analyzes, namely: (1) effectiveness test used *one sample t-test*, (2) difference test used *MANOVA*, and (3) comparison among the learning used *t-Benferroni test*. Normality test and homogeneity test were assumption test which had to be fulfilled before inferential analysis. Normality test used *Mahalanobis distance* and homogeneity test used *Box's M* at the significance level of 5%.

C. Findings and Discussion

Learning process in both classes (experiment class and control class) during eight meetings was conducted per lesson plan. However, during the learning process, there were some obstacles such as time allocation, students' condition, facilities in class so that there were some steps in learning were not implemented. However, overall, the researcher can be concluded that MMP-PS and MMP in both classes were well done.

In this study, the result of analysis data consisted of two parts namely data description and inferential analysis to test the hypothesis. Data on mathematical reasoning ability and self-efficacy is shown in Table 5 and Table 6.

Table 5. Data description of mathematical reasoning ability

Description	MMP-PS		MMP	
	Pretest	Posttest	Pretest	Posttest
Mean	3,64	11,70	2,78	11,13
Standard Deviation	2,58	2,15	1,17	2,83
Maximum score ideal	15	15	15	15
Minimum score ideal	0	0	0	0
Maximum score	9	15	6	15
Minimum score	1	8	1	5

Table 6. Data description of self-efficacy

Description	MMP-PS		MMP	
	Pretest	Posttest	Pretest	Posttest
Mean	91,08	107,4	89,43	94,30
Standard Deviation	14,28	14,99	15,90	11,67
Maximum score ideal	150	150	150	150
Minimum score ideal	30	30	30	30
Maximum score	116	137	120	114
Minimum score	68	86	61	73

Table 5 and Table 6 show that there is an increase in mathematical reasoning ability and self-efficacy of the student after treatment in both classes. The average score of mathematical reasoning ability before being given treatment in experiment class (MMP-PS) and control class

(MMP) is 3,64 and 2,78, but after being given treatment, mathematical reasoning ability of student is 11,7 and 11,13. It showed that there was an increase in average score mathematical reasoning ability in experiment class of 8,06 and control class of 8,35.

Meanwhile, the average score of self-efficacies before being given treatment in experiment class (MMP-PS) and control class (MMP) is 91,08 and 89,43, but after being given treatment, self-efficacy of the student is 107,4 and 94,30. It showed that there was an increase of average score self-efficacy of the student in experiment class of 16,32 and control class of 4,87. Then data of self-efficacy were converted into qualitative criteria based on Table 1. The result of conversion can be seen in Table 7.

Table 7. Student's self-efficacy criteria

Interval	**Criteria**	**MMP-PS**				**MMP**			
		Pre		**Post**		**Pre**		**Post**	
		N	**%**	**N**	**%**	**N**	**%**	**N**	**%**
X > 120	Very High	0	0	5	20	0	0	0	0
100 < X ≤ 120	High	5	20	12	48	5	21,74	7	30,43
80 < X ≤ 100	Moderate	14	56	8	32	15	65,22	15	65,22
60 < X ≤ 80	Low	6	24	0	0	3	13,04	1	4,35
X ≤ 60	Very Low	0	0	0	0	0	0	0	0

Table 7 showed that 19 students experienced an increase self-efficacy in experiment class, while in control class 6 students experienced self-efficacy increased and there was one student who experienced decreased self-efficacy score.

Next step is inferential analysis, but before doing this analysis, there is assumption test must be fulfilled namely normality test and homogeneity test. The result of normality and homogeneity test can be seen in Table 8 and Table 9 below.

Table 8. Normality test

	MMP-PS		**MMP**	
	Pre	**Post**	**Pre**	**Post**
Percentage of Students with the score $\boldsymbol{d_i}^2 < \boldsymbol{x^2 0,5(2)}$	52%	52%	47,83%	52,17%

Based on Table 8, multivariate normality test used *Mahalonobis distance* because there was about 50% of student got a score less than *chi-square* $x^2 0{,}5(2)$.

Table 9. Homogeneity test

Data	**Box's M**	**F**	**df1**	**df2**	**p-value**
Pretest	18,776	2,906	3	15025,653	0,08
Posttest	7,246	2,301	3	15025,633	0,075

Based on Table 9, it is known that p-value is greater than 0,05. It means that the assumption of homogeneity has been fulfilled before and after treatment was applied in both classes. Because the assumption of normality and homogeneity test have been fulfilled, data analysis can continue to inferential analysis.

The result of effectiveness test for MMP-PS and MMP can be seen in Table 10.

Table 10. The result of one sample t-test

Test Value of MRA = 11 and SE = 102				
Treatment	**T**		**p-value**	
	MRA	**SE**	**MRA**	**SE**
MMP-PS	3,99	1,80	0,001	0,042
MMP	1,91	-3,16	0,005	0,069

MRA = mathematical reasoning ability; SE = self-efficacy

Based on Table 10, experiment class which was taught using MMP-PS, was effective based on mathematical reasoning ability and self-efficacy because the value of t in the dependent variable (MRA and SE) is greater than $t_{0,05;24} = 1,71$ with p-value respectively is less than 0,05, so alternative null hypothesis is rejected. Meanwhile, in control class obtained t-value is 1,91 for MRA and -3,16 for SE with the p-value 0,005 and 0,069 respectively. T-table used in this condition was $t_{0,05;22} = 1,72$. It can be concluded that MMP learning was effective based on mathematical reasoning ability, but MMP learning was not effective viewed from self-efficacy of the student.

The statistical results above show that MMP-PS and MMP are effective based on student's mathematical reasoning ability. Steps in MMP-PS and MMP are same, however, at the second step (mathematical concepts) step, MMP-PS learning triggers student's activities to find mathematical concepts independently through problem-solving. It is in line with the results of Rivai and Surya's study (2017) which stated that students who were taught using Missouri mathematics project were able to fulfill the achievement of mathematical reasoning indicators well.

The first step in Missouri mathematics project learning is to provide stimulus to students to provoke student's knowledge related to the concepts learned through review activities. In this activity, students were invited to recall the mathematics topics that they had learned related to the concepts to be studied. Furthermore, students were also invited to observe objects around them that relate to the concept.

The second step is the development of material (mathematical concept). This step has an essential role in improving students' mathematical reasoning ability. In this step, students work in groups of 4-5 people. Then each group of students received the worksheet which contained many mathematical problems to solve that were relevant to improve student's mathematical reasoning ability. For example, the aspect of the ability to find patterns is improved through activities available at the worksheet. The activity begins with observing the problem and solving the problem students must find a particular mathematical concept that is relevant to solve it. The ability to make conjecture is trained through identifying information or understanding problems. Next, after students understand what they are looking for in the problem, together with their group, students plan to solve the problem. Finally, to practice the ability to give evidence, students provide explanations by solving the problems given. Not only that, students are given the opportunity to provide explanations related to their answers to other groups through class discussions. With such activities, students are active and provide opportunities to explore their ability to solve mathematical problems, so that they are accustomed to solving problems through investigation.

The development of concept phase is carried out by the students themselves on the small heterogeneous group. Collaboration in a group encourages students to share information and help each other, especially to a friend who experienced difficulty (Huda, 2017). In work together in groups, students have their respective responsibility and have to contribute to the achievement of group goals. In this case, each student has the opportunity to express their idea or opinion about the concept that will be found or problem that will be solved. Present opinions by orally show student's skills in thinking and composing the idea are to be easily understood by other students. In the class discussion, the student who gets the assignment presents the results of the group's discussion when he explains to other groups. The student is also required to explain properly so that other groups understand easily. The result is not directly the student can better understand the concept the mathematics and its usefulness to solve the problem (Huda, 2017).

In MMP learning, students' mathematical reasoning ability also increase. It is because some projects completed by students refer to the sample questions presented in the LKPD. At the material development step, students were taught using MMP independently learn mathematical concepts through examples that have been available at the LKPD. After students understand the concept, they begin to apply the concept to solve problems with different levels of difficulty to train student’s mathematical reasoning ability. If in the group, students experience difficulties in understanding or solving problems, then the teacher is tasked with providing guidance in the form of asking questions to students who characterize students to answer their difficulties. In line with the results of research by Melianingsih and Sugiman (2015), Sardin (2015), and Falach (2016) showed that there was an increase in mathematical reasoning ability of students who were taught using problem-solving approach. In short, students will try to find mathematical concepts through problem solved.

In the case of self-efficacy, there are differences between students who were taught with MMP-PS and MMP. MMP-PS was effective, but MMP was not effective. Ineffective MMP based on self-efficacy is not line with Rembely (2013) that learning based project can improve interest and belief in their ability to solve the problem. In the experiment class, MMP learning was combined with a problem-solving approach. Another study by Martyanti (2016) showed that cooperative learning (problem-solving learning) was effective in increasing students’ self-confidence in learning mathematics. It is known that self-confidence is part of self-efficacy. In concept development and controlled exercises activities which presented in a form problem solving requires students to work together in the group. This condition allows students to express their opinions and exchange ideas in solving the problem. Besides, working in groups allows students to obtain information and more new knowledge and can increase students' understanding of the particular mathematical concept (Van de Walle, 2007). As a result, when students solve the problems independently, they are sure or believe of their ability because they have obtained sufficient knowledge to solve the other problem.

Implementation of MMP in the control class, there were some students when working in the group were not active and did not dare to express their opinion in solving the problem. Besides that, MMP is not accompanied by class discussion, so students were not trained to express their opinions. In this condition, there were also some students who did not dare to ask questions to other students and teacher even though they did not understand the subject they had learned in group discussion. It indicates MMP is not effective concerning self-efficacy.

We concluded that the main factor that distinguishes the results of research in MMP-PS and MMP classes based on self-efficacy lies in the material development and discussion step. MMP-PS provides a full activity to students to express their opinions in the form of group discussions and then proceed with class discussions. It makes each student have their responsibility to understand each subject, and they try when other groups easily understand the presentation of mathematics concept. Such activities have a significant role in increasing students' confidence in their ability to understand the material. However, MMP is only limited to group discussions, so students are not very active with other groups in sharing their opinions so that the intensity of their activities is limited. Students taught by using MMP will also be afraid to express their opinions or ask questions. When they are asked to solve problems in front of the class, they do not want to go forward because they still feel unsure of their abilities even though some of them have enough knowledge to solve the problem.

From the data analysis, we found that the ability to give a reason for the solution is the aspect that had significant increase compared to others aspect of mathematical reasoning ability. The aspect of giving a reason for the solution increase to 2,68 in experiment class and 2,96 in control class. An example of student work on this aspect is presented in Figure 1. The problem given in this case is as follows

The distance of the center point of circle A and circle B is 15 cm. if the radius of the two circles is 5 cm and 4 cm respectively, investigate the tangent length of fellowship in the circles is 12 cm.

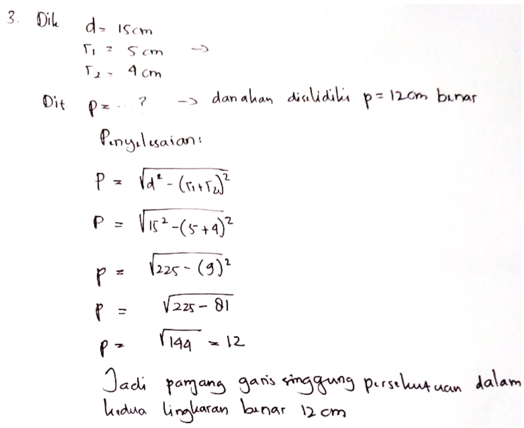


Figure 3. Student's work on the given reason for the solution aspect

Based on Figure 3 above shows that student could understand well the purpose of the problem. It can be seen from the students being able to rewrite all information needed to solve the problem. The student also looks fluent to write mathematics concept in the form of the right formula used to solve the problem. In the end, the student writes a conclusion to confirm that his answer is following the solution given to the problem.

According to Table 8 and Table 9, analysis of the average difference in post-test result MMP-PS and MMP can be done using Manova. The result of Manova can be seen in Table 11 below.

Table 11. Result of manova test

Data	F	p-value (Hotelling's trace)
Posttest	6,347	0,004

Result of Manova using posttest data showed that F-value = 6,347 is greater than $F_{(0,05;2;46)} = 3{,}199$ with p-value = 0,004 is less than 0,05, so that alternative null hypothesis is rejected. So, it can be concluded that there is a difference in mathematical reasoning ability and self-efficacy of students after treatments were applied. Therefore, the researcher analyzed to find which learning was better based on mathematical reasoning ability and self-efficacy of students. T-Benferroni test was used to confirm the hypothesis. The result of t-Benferroni test is presented in Table 12.

Table 12. Result of t-Benferroni test

Data	Variable	T	p-value
Posttest	MRA	0,82	0,419
	SE	3,36	0,002

The result of t-Benferroni test showed that in variable mathematical reasoning ability obtained the value of t = 0,82 is less than $t_{(0,017;46)} = 2{,}48$ with p-value = 0,419 is greater than 0,05, so it can be concluded that MMP-PS was not more superior than MMP viewed from mathematical reasoning ability after treatments were applied. Meanwhile, analysis result obtained value of t = 3,36 is greater than $t_{(0,017;46)} = 2{,}48$ and p-value = 0,002 is less than 0,05. It can be concluded that MMP-PS was better than MMP viewed from self-efficacy after treatments were applied.

One of the factors which make MMP-PS more superior than MMP based on self-efficacy is the intensity of student activity and allowing student self-efficacy to increase. In the group discussion, students generally tend to be more interested in their friend's ideas, so they do not feel embarrassed or afraid when they want to express their opinions. Also, with the discussion, each student will get much input and if anyone has not understood the concept so the other student will help. Not only for students but also class discussion activities will help the teacher to get enough information about the condition of students regarding their thinking skill to solve the problem so that, the teacher can give feedback when students experience errors in understanding the concept. Thus, these activities make students not only skilled in answering questions but also students can give arguments regarding their answers. As a result, when students are working on the problem independently, they will feel confident in their abilities because they have enough knowledge to solve the problem.

The results of this study are expected to be able to explain Missouri mathematics project learning which is combined with a problem-solving approach. This study focused on the effectiveness of MMP-PS learning in mathematics learning toward student's mathematical reasoning ability and self-efficacy. The results showed that students' mathematical reasoning ability was better and students' self-efficacy increased after MMP-PS learning was applied. MMP-PS learning is effective toward mathematical reasoning ability and self-efficacy affected by several factors including the concept development phase because the students found mathematical concept through problem-solving. It means that students were directly involved starting from understanding/identifying the problem, making plans, and finally finding the mathematical concept. The problem used in this case was a real-world problem. Then students were also given controlled exercise, where the problem was given in the form of non-routine problems that were different from the problems that have been solved by the student. It was proven to improve student's reasoning abilities and self-efficacy in learning mathematics. Therefore, Missouri mathematics project-problem solving learning (MMP-PS) can be alternative learning in improving and developing mathematical reasoning abilities and self-efficacy of students in mathematics learning.

The study we conducted has several limitations that cannot be completely covered. *Firstly*, the lesson in this study only consisted of eight meetings then we assessed mathematical reasoning ability and self-efficacy. To find out how effective the learning is in improving mathematical reasoning ability and self-efficacy require a relatively long time. *Secondly*, the mathematics topic is limited to the circle, so generalization of results becomes limited. Moreover, the last, there were meetings with no observers which implement learning cannot be adequately observed.

D. Conclusion

This article thoroughly describes that MMP-PS learning could improve mathematical reasoning ability and self-efficacy. The result of the inferential statistical test shows the increase in almost every indicator of mathematical reasoning ability and each dimension of self-efficacy. However, the ability of students taught by MMP learning increase only on mathematical reasoning abilities since the lack of students' involvement in exploring their abilities which influence self-trust in their abilities. Besides, there is a difference in mathematical reasoning ability and self-efficacy among students are taught using MMP-PS and MMP, but overall MMP-PS is better than MMP. The relation between mathematical reasoning ability and student's self-efficacy or how the level of self-efficacy of students influences mathematical reasoning ability in MMP-PS learning could be further research.

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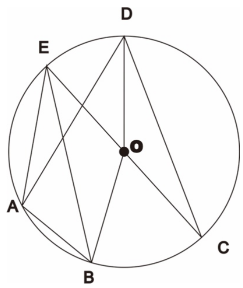
Appendix

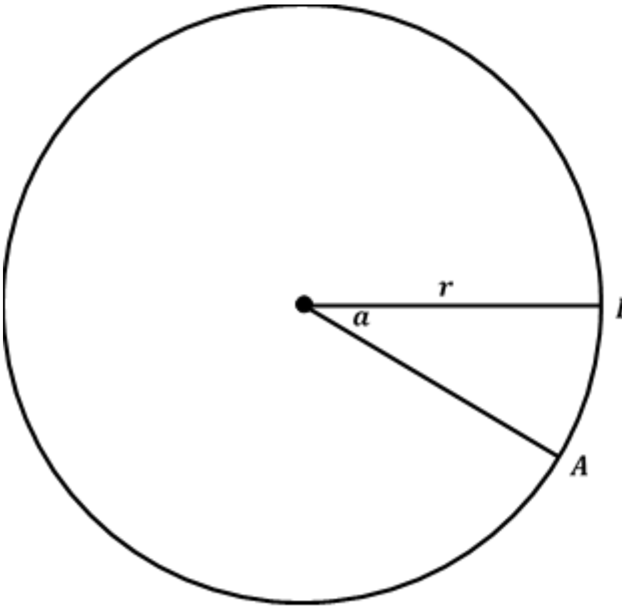
Mathematical Reasoning Ability Test

1. Look at the following picture.

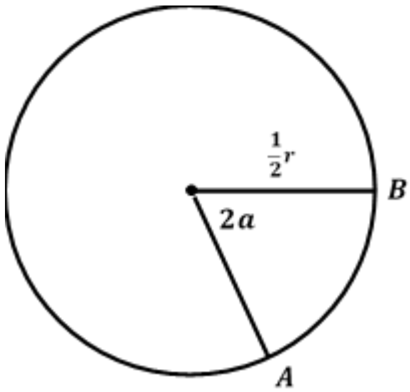


a. Mention the angles that are equal to the size of $m\angle ADC$!
b. Explain your reason!

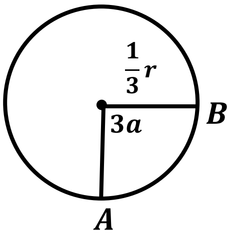
2. Look pattern below. The center angle of the circle is $na \leq 360^o$. If the picture continues, determine the arc length AB in the sixth picture!



Gambar 1

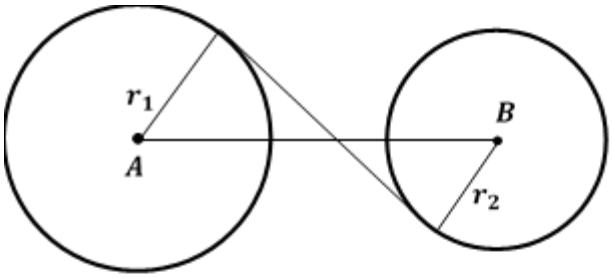


Gambar 2



Gambar 3

3. Consider the following picture. The distance of the center of circle A and circle B is 15 cm. If the radius of the two circles is 5 cm and 4 cm, investigate whether the length of the tangent line in the two circles is 12 cm!



4. Look the following pictures pattern! The length of the square side is 24 cm, and each circle coincides. If the third picture continues, determine the area of the unshaded area in the sixth picture!